Texas Instruments
TI-99/4 Home Computer

Diskette Software

Model PHD 5006

Math Routine Library

Five powerful programs which enable you to perform sophisticated mathematical analyses.

- **FOURIER SERIES — Computes Fourier coefficients from an explicit function or a series of discrete values.**
- **ORDINARY DIFFERENTIAL EQUATIONS — Solves nth degree equations using the Runge-Kutta fourth-order numerical method.**
- **FUNCTION ANALYSIS — Generates graphs and tables, and finds zeros for a function you define.**
- **BASES, PRIMES, HYPERBOLICS — Performs base conversions, computes prime factors, and calculates hyperbolic and inverse hyperbolic functions.**
- **SIMULTANEOUS EQUATIONS AND MATRIX INVERSION — Computes the determinant and inverse of a matrix and solves up to ten simultaneous equations with ten unknowns.**

Designed for use with the TI-99/4 Home Computer and the TI Disk Memory System (TI Disk Drive Controller and TI Disk Memory Drive — sold separately).

As this manual was designed for the U.S. market, the warranty conditions described herein are not applicable in the U.K. The only valid Guarantee Conditions are those set forth in the "Users Reference Guide" accompanying the Home Computer.

Author : Texas Instruments

Language : TI-99/4 BASIC

Hardware : TI-99/4 Home Computer
Disk Controller and Drive or Cassette Recorder
Thermal Printer (Optional)
RS232 Interface and Compatible Printer (Optional)

Media : Diskette or Cassette

The Math Routine Library is a collection of programs which enable you to perform sophisticated mathematical analyses and use complex functions in a simple, straightforward manner. With these programs you can compute the coefficients in a Fourier series, solve ordinary differential equations, integrate and differentiate mathematical functions, evaluate hyperbolic functions, solve simultaneous equations, and perform many other mathematical computations.

Each program includes a predefined function. You may use the program to analyze this function or enter one of you own. In either case, the computer asks you for the values it needs to perform its computations.

The Fourier Series program computes the Fourier coefficients, A_k and B_k, of a Fourier series. You may use the program either by defining an explicit function, F, or by entering discrete values for the function. When you specify the values for the period, T, and the number of coefficients, N, the computer calculates and displays the values of the coefficients.

The Fourier series is an expansion series used in the mathematical analysis of complex waveforms. It enables the function under consideration to be resolved into a fundamental frequency, plus a finite number of terms involving its harmonics. Fourier series apply to periodic functions, which are defined as those that repeat themselves every T seconds. In order for the expansion of the function, F(x), to be valid, certain conditions must be met. For example, F(x) must be finite and single-valued. It must have a finite number of maximum and minimum values within one period, and it must have a finite number of discontinuities. When these conditions are met, the function may be represented, except at the discontinuities, by a series of simple harmonic functions in which the frequencies are integral multiples of the fundamental frequencies. In the Fourier series, all the terms of the series are added, and when the number of terms, n, becomes infinitely great, the sum approaches the value of F(x).

In the actual expansion of the function, F(x) is given by:

$$F(x) = A_0/2 + \sum_{K=1}^{\infty} G_K(x), \quad \text{where}$$

$$G_K = A_K \ COS(K\omega x) + B_K \ SIN(K\omega x)$$

$$\omega = 2\pi/\tau, \quad \text{and}$$

$$\tau = \text{Period of F}$$

In this expression, the coefficients for the sine and cosine terms are given by:

$$A_K = 2/\tau \int_0^T F(x)COS(K\omega x)dx$$

$$B_K = 2/\tau \int_0^T F(x)SIN(K\omega x)dx$$

STEP 1: If the computer is not already in the BASIC mode, select TI BASIC. To load the program from a diskette, insert the diskette into the disk drive, type

OLD DSK1.FOURIER

and press ENTER.

To load the program from a cassette tape, refer to the "Loading Cassettes" section in this manual for instructions on determining the program's position on the cassette tape. Then insert the cassette into the recorder, type

OLD CS1

and press ENTER. The computer then displays directions for loading the tape. Refer to "Loading Cassettes" if you have difficulty in loading the program from the cassette.

STEP 2: When the cursor appears, type RUN, and press ENTER. When the FOURIER SERIES title screen appears, press any key. Then select one of the following options:

1 DESCRIPTION
2 COEFFICIENTS
3 EXIT

OPTION 1: DESCRIPTION
If you select option 1, the computer displays a mathematical description of the Fourier series. When you are ready to continue, press any key. The computer then automatically proceeds to option 2, COEFFICIENTS.

OPTION 2: COEFFICIENTS
If you select option 2, the computer displays the formulas for the coefficients in the Fourier series and tells you that these coefficients are now being calculated. You can calculate coefficients for a predefined function, or you may

enter discrete values for the function. To calculate the coefficients for the predefined function, press 1. To input discrete values for the function, press 2.

NOTE: For information on defining the function for which the program computes Fourier coefficients, see the "Modifications" section.

To perform calculations with the predefined function, press 1. The computer asks you for the value for the period, T, and the number of coefficients, N, to be calculated. Type the values, pressing ENTER after each one. The computer then calculates the values for A_k and B_k. Press SHIFT V (PROC'D) to return to the selection list. To calculate coefficients for other values of T or N, press SHIFT R (REDO), and the computer again asks for T and N.

If you have chosen to input discrete values of the function by pressing 2, the computer first asks you for the period, T, and the number of values for the function. Then you are asked to enter each of the discrete values. After you have entered all of the values, the computer calculates and displays the coefficients for each of the discrete values. To return to the selection list, press SHIFT V (PROC'D). To calculate coefficients for other discrete values of the function, press SHIFT R (REDO).

NOTE: The values for the coefficients are rounded to five decimal places. For long lists of coefficients, press the space bar to stop and start the display of the coefficients.

OPTION 3: EXIT
To return to TI BASIC, select option 3 from the selection list, or press SHIFT C (CLEAR).

This example demonstrates entering 12 discrete values for a function and computing the Fourier coefficients. After you load and run the program, select COEFFICIENTS by pressing 2. Next press 2 to input discrete values. Then enter the information shown here:

```
PERIOD T ? 6.283
HOW MANY F VALUES ? 12
F VALUE 1 ? 2
F VALUE 2 ? 3.2
F VALUE 3 ? 3.7
F VALUE 4 ? 2.5
F VALUE 5 ? 1.2
F VALUE 6 ? 1.5
F VALUE 7 ? 2.7
F VALUE 8 ? 2.4
F VALUE 9 ? 0
F VALUE 10 ? -3.3
F VALUE 11 ? -3.3
F VALUE 12 ? 0
```

After you enter the last F-value, the computer calculates the coefficients. As mentioned previously, the computer rounds the values to five decimal places.

```
A0=2.43333
B0=0

A1=-1.25056
B1= 1.71929

A2=-0.55
B2= 1.90526

A3= .75
B3= .18333

A4= .58333
B4= .05774

...
...
...

A11=-1.25056
B11=1.71929

A12= 2.43333
B12= 0
```

To return to the selection list, press any key.

To define the function for which the computer calculates coefficients, modify line 150 of the program using the following procedure:

STEP 1: After you have loaded the program as described in STEP 1 of the "User Instructions," press SHIFT C (CLEAR) to enter the TI BASIC mode.

STEP 2: Type "150" and press SHIFT X (DOWN).

STEP 3: To define a new function, F(X), press SHIFT D (RIGHT) to move the cursor to the right of the equal sign, and type the new function over the old one. Use SHIFT F (DELETE) to delete the remainder of the previous function, if any.

STEP 4: When you are sure you have typed the new function correctly, press ENTER. When the cursor reappears, type RUN and press ENTER. The computer then displays the FOURIER SERIES title screen.

Ordinary Differential Equations enables you to solve ordinary differential equations with initial conditions. The computer gives you a mathematical description for the reduction of Nth order differential equations to a system of N first-order differential equations. When you enter the number of initial conditions, the increment, T, the number of increments, and the initial conditions, the computer calculates and displays the values of the functions for each increment.

An Nth degree ordinary differential equation with initial conditions can be solved with the Runge-Kutta 4th order numerical method. (Reference: Hornbeck, Robert W. <u>Numerical Methods</u>. New York: Quantum Publishers, Inc., 1975.) The iteration formula is:

$$y_{j+1} = y_j + \Delta t[f(y_j,t_j)/6 + f(y^*{}_{j+1/2},t_{j+1/2})/3 + f(y^{**}{}_{j+1/2},t_{j+1/2})/3 + f(y^*{}_{j+1},t_{j+1})/6]$$

where

$$y^*{}_{j+1/2} = y_j + \Delta t[f(y_j,t_j)]/2$$

$$y^{**}{}_{j+1/2} = y_j + \Delta t[f(y^*{}_{j+1/2},t_{j+1/2})]/2$$

$$y^*{}_{j+1} = y_j + \Delta t[f(y^{**}{}_{j+1/2},t_{j+1/2})]$$

It is assumed that any Nth-order differential equation can be reduced to a system of N first-order differential equations of the form:

$$dy_1/dt = f_1(y_1, y_2, \ldots, y_n, t)$$

$$dy2/dt = f_2(y_1, y_2, \ldots, y_n, t)$$

$$\vdots$$

$$dy_n/dt = f_n(y_1, y_2, \ldots, y_n, t)$$

subject to the initial conditions:

$$y_1(0) = y_{10}$$

$$y_2(0) = y_{20}$$

$$\vdots$$

$$y_n(0) = y_{n0}$$

You can define as many as ten functions to be used in solving the equations. For information on defining functions, see the "Modifications" section.

STEP 1: If the computer is not already in the BASIC mode, select TI BASIC. To load the program from a diskette, insert the diskette into the disk drive, type

OLD DSK1.ODES

and press ENTER.

To load the program from a cassette tape, refer to the "Loading Cassettes" section in this manual for instructions on determining the program's position on the cassette tape. Then insert the cassette into the recorder, type

OLD CS1

and press ENTER. The computer then displays directions for loading the tape. Refer to "Loading Cassettes" if you have difficulty in loading the program from the cassette.

STEP 2: When the cursor appears, type RUN, and press ENTER. When the ORDINARY DIFFERENTIAL EQUATIONS title screen appears, press any key. Then select one of the following options:

1 DESCRIPTION
2 COMPUTATIONS
3 EXIT

OPTION 1: DESCRIPTION
If you select option 1, the computer displays a mathematical description for the reduction of Nth-order differential equations. When you are ready to continue, press any key. The computer automatically proceeds to option 2, COMPUTATIONS.

OPTION 2: COMPUTATIONS
If you select option 2, enter the number of functions (n), the increment (t), and the number of increments. You are then asked to enter the initial values, $y_{1(0)}$, $y_{2(0)}$, ... , $y_{n(0)}$. After you enter the initial conditions, the computer calculates and displays the y_j values for each t increment.

NOTE: Press the space bar to stop the display of the values.

To return to the option selection list, press SHIFT V (PROC'D). If you want to solve other equations or the same equations with different initial conditions, press SHIFT R (REDO).

OPTION 3: EXIT
To return to TI BASIC, select option 3 from the selection list, or press SHIFT C (CLEAR).

Given the initial value problem:

$$d^2y/dt^2 + 2dy/dt + 4y = 0$$
$$y(0) = 2, \quad dy/dt(0) = 0$$

This problem is converted to the first order system:

$$dy_2/dt = -2y_2 - 4y_1$$
$$dy_1/dt = y_2$$

with the initial conditions: $y_2(0) = 0$, $y_1(0) = 2$.

To solve this problem, select COMPUTATIONS as described in STEP 2 of the "User Instructions" and enter the following information

```
NUMBER OF INTITIAL CONDITIONS (N) ? 2
T INCREMENT ? .1
NUMBER OF INCREMENTS ? 4
ENTER INITIAL CONDITIONS:
  Y1(0) = ? 2
  Y2(0) = ? 0
```

The computer then calculates the y_j values rounded to five decimal places, as shown here:

```
T = 0
Y1(0) = 2
Y2(0) = 0

T = .1
Y1(.1) = 2.0527
Y2(.1) = .081

T = .2
Y1(.2) = 2.02168
Y2(.2) =-.67434

T = .3
Y1(.3) = 1.92322
Y2(.3) =-1.2684
```

To return to the selection list, press any key.

To define the functions F1, F2,..., Fn with which the program operates, modify the program using the following procedure:

STEP 1: After you have loaded the program as described in STEP 1 of the "User Instructions," press SHIFT C (CLEAR) to enter the TI BASIC mode.

STEP 2: Type "LIST 140-230" and press ENTER. The computer then lists the following program lines:

```
140 DEF F1=Y2
150 DEF F2=-2*Y2-4*Y1
160 DEF F3=0
      .
      .
      .
230 DEF F10=0
```

STEP 3: After you have decided which one of the functions you wish to modify, type the line number for that particular function and press SHIFT X (DOWN).

STEP 4: To define a new function, F(X), press SHIFT D (RIGHT) to move the cursor to the right of the equal sign, and type the new function over the old one. Use SHIFT F (DELETE) to delete the remainder of the previous function, if any.

STEP 5: When you are sure you have typed the new function correctly, press ENTER. You may define as many as 10 functions in the original program or you may add additional ones on the unused lines between line 140 and 230. When you have defined all the functions to be used, type RUN and press ENTER. Then the computer displays the ORDINARY DIFFERENTIAL EQUATIONS title screen.

Function Analysis allows you to graph, generate a table of x and y values, integrate, differentiate, and find zeros for a predefined function or a function you define.

STEP 1: If the computer is not already in the BASIC mode, select TI BASIC. To load the program from a diskette, insert the diskette into the disk drive, type

OLD DSK1.FUNCTION

and press ENTER.

To load the program from a cassette tape, refer to the "Loading Cassettes" section in this manual for instructions on determining the program's position on the cassette tape. Then insert the cassette into the recorder, type

OLD CS1

and press ENTER. The computer then displays directions for loading the tape. Refer to "Loading Cassettes" if you have difficulty in loading the program from the cassette.

STEP 2: When the cursor appears, type RUN, and press ENTER. When the FUNCTION ANALYSIS title screen appears, press any key. The computer tells you that the function, F, may be defined at line 140. To define the function, see the "Modifications" section. Press any key, and select one of the following options:

1 GRAPH
2 TABLE
3 INTEGRATE
4 DIFFERENTIATE
5 FIND ZEROS
6 EXIT

OPTION 1: GRAPH
If you select option 1, the computer asks you for the interval (A, B) on which the function, F, is to be graphed. After you enter A and B, the computer generates and displays a graph of the function within the specified interval. The computer also displays the maximum and minimum values for the function within the interval. To return to the selection list, press any key.

OPTION 2: TABLE
If you select option 2, the computer asks you for the interval (A, B) and a step increment, H. You then have a choice of printing the table on the screen, the Thermal Printer, or another type of output device. To name the output device, refer to the RS232 manual for instructions. When you make your selection, the computer generates and displays or prints a table of x values and the corresponding values of F(x). To return to the selection list, press any key.

OPTION 3: INTEGRATE
If you select option 3, the computer uses Simpson's rule to calculate the Nth approximation of the integral F(x) on the interval (A, B). You are asked to enter A, B, and N. The computer calculates and displays the Nth approximation of the integral. Press SHIFT V (PROC'D) to return to the selection list. To obtain other integrals, press SHIFT R (REDO), and the computer again asks for A, B, and N.

OPTION 4: DIFFERENTIATE
If you select option 4, the computer calculates the Nth approximation of the derivative F'(X) with the difference quotient:

$$F'(X) = \frac{F(X+1/n) - F(X)}{1/n}$$

You are asked to enter values for X and n, and the computer then calculates and displays the Nth approximation of the derivative. To return to the selection list, press SHIFT V (PROC'D). To obtain approximations at other points, press SHIFT R (REDO), and the computer again asks for X and n.

OPTION 5: FIND ZEROS
If you select option 5, you can find the zeros of the function, F, by a Newton method. You are asked to enter an interval (A, B) such that F(A) and F(B) differ in

sign. If the interval you enter does not satify this condition, you are asked to enter another interval. Once a legitimate interval is entered, the algorithm goes through 50 iterations of bisecting the interval to pinpoint the zero. The resulting zero, X, and F(X) are then displayed. Press SHIFT V (PROC'D) to return to the selection list. To find zeros for a different interval, press SHIFT R (REDO).

OPTION 6: EXIT
To return to TI BASIC, select option 6 from the selection list or press SHIFT C (CLEAR).

To define the function which the computer analyzes, modify line 140 of the program using the following procedure:

STEP 1: After you have loaded the program as described in STEP 1 of the "User Instructions," press SHIFT C (CLEAR) to enter the TI BASIC mode.

STEP 2: Type "140" and press SHIFT X (DOWN).

STEP 3: To define a new function, F(X), press SHIFT D (RIGHT) to move the cursor to the right of the equal sign, and type the new function over the old one. Use SHIFT F (DELETE) to delete the remainder of the previous function, if any.

STEP 4: When you are sure you have typed the new function correctly, press ENTER. When the cursor reappears, type RUN and press ENTER. The computer then displays the FUNCTION ANALYSIS title screen.

BASES, PRIMES, AND HYPERBOLICS can be used to convert a number from one base to another, to calculate prime factors, and to calculate the value of hyperbolic or inverse hyperbolic functions.

In converting a number, any base from 2 through 36 can be used. The number to be converted can be up to 100 characters including a decimal point. However, because TI BASIC can only handle numbers from 10^{-128} through 10^{126}, the program does not work for numbers outside this range. The conversion is accurate to 12 significant decimal digits for the integer portion of the number. The fractional part has the same accuracy but is truncated to seven characters.

A second option in the program calculates and displays the prime factors for positive integers.

Two other options enable you to calculate and display the hyperbolic or inverse hyperbolic functions at a point which you enter. These include SINH, COSH, TANH, COTH, CSCH, and SECH for the hyperbolics and $SINH^{-1}$, $COSH^{-1}$, $TANH^{-1}$, $COTH^{-1}$, $CSCH^{-1}$, and $SECH^{-1}$ for the inverse hyperbolics.

STEP 1: If the computer is not already in the BASIC mode, select TI BASIC. To load the program from a diskette, insert the diskette into the disk drive, type

OLD DSK1.B/P/H

and press ENTER.

To load the program from a cassette tape, refer to the "Loading Cassettes" section in this manual for instructions on determining the program's position on the cassette tape. Then insert the cassette into the recorder, type

OLD CS1

and press ENTER. The computer then displays directions for loading the tape. Refer to "Loading Cassettes" if you have difficulty in loading the program from the cassette.

STEP 2: When the cursor appears, type RUN, and press ENTER. When the BASES, PRIMES, AND HYPERBOLICS title screen appears, press any key. Then select one of the following options:

1 BASE CONVERSION
2 PRIME FACTORIZATION
3 HYPERBOLICS
4 INVERSE HYPERBOLICS
5 EXIT

OPTION 1: BASE CONVERSION
If you select option 1, the computer asks you for the two bases, B1 and B2. These bases may be from 2 to 36. Enter the number in base B1, which must be positive and cannot be in scientific notation. The computer then converts and displays the number in base B2. To return to the selection list, press SHIFT V (PROC'D). To make other base conversions, press SHIFT R (REDO).

OPTION 2: PRIME FACTORIZATION
If you select option 2, enter a number, N. The computer then calculates and displays the prime factors for that number. To return to the selection list, press SHIFT V (PROC'D). To obtain the prime factors of another number, press SHIFT R (REDO).

OPTION 3: HYPERBOLICS
If you select option 3, enter a number, X. The computer calculates and displays the hyperbolic functions for that number. To return to the selection list, press SHIFT V (PROC'D). To evaluate the hyperbolic functions for another value of X, press SHIFT R (REDO).

OPTION 4: INVERSE HYPERBOLICS
If you select option 4, enter a value for X, the number to be evaluated. The computer then calculates and displays the inverse hyperbolic functions for that value. To return to the selection list, press SHIFT V (PROC'D). To evaluate the inverse hyperbolic function for another value of X, press SHIFT R (REDO).

OPTION 5: EXIT
To return to TI BASIC, select option 5 from the selection list or press SHIFT C (CLEAR).

Simultaneous Equations and Matrix Inversion performs a variety of calculations including displaying and inverting a matrix, computing determinants, and solving simultaneous equations.

You may enter and display up to a 10 by 10 matrix. The program uses the Gramm-Schmidt procedure to compute both the determinant and the inverse of the matrix. (Reference: Birkhoff, Garrett, and Saunders MacLane. A Survey of Modern Algebra. New York: McMillan Co., 1953.) Additionally, you can use the program to solve up to 10 simultaneous equations with 10 unknowns.

STEP 1: If the computer is not already in the BASIC mode, select TI BASIC. To load the program from a diskette, insert the diskette into the disk drive, type

OLD DSK1.SIMEQ

and press ENTER.

To load the program from a cassette tape, refer to the "Loading Cassettes" section in this manual for instructions on determining the program's position on the cassette tape. Then insert the cassette into the recorder, type

OLD CS1

and press ENTER. The computer then displays directions for loading the tape. Refer to "Loading Cassettes" if you have difficulty in loading the program from the cassette.

STEP 2: When the cursor appears, type RUN, and press ENTER. When the SIMULTANEOUS EQUATIONS AND MATRIX INVERSION title screen appears, press any key. Then select one of the following options:

1 ENTER MATRIX
2 DISPLAY MATRIX
3 CHANGE MATRIX
4 INVERT MATRIX
5 COMPUTE DETERMINANT
6 SOLVE EQUATIONS
7 REDIRECT OUTPUT
8 EXIT

OPTION 1: ENTER MATRIX
You must select option 1 first because options 2 through 8 are not available until you enter the matrix. The matrix must be square. For example, a second order matrix has two rows and two columns; a third order matrix has three rows and three columns. When the computer asks you for the size of

the matrix, enter its order. Then enter the elements of row 1 separated by spaces; enter the elements of row 2 separated by spaces. Continue to enter the elements in rows until the matrix is complete. Then press ENTER and the computer returns to the selection list.

OPTION 2: DISPLAY MATRIX
If you select option 2, the computer displays the matrix you have entered so that you can see if it is correct. Press any key to return to the selection list.

OPTION 3: CHANGE MATRIX
If you selct option 3, the computer asks you for the row and column of the element you wish to change. Enter the row number and column number separated by a comma. The computer then displays the old value for that element and asks you to enter the new one. After you enter the new element, press any key to return to the selection list.

OPTION 4: INVERT MATRIX
If you select option 4, the computer calculates and displays the determinant and the inverse of the matrix. Press any key to return to the selection list.

OPTION 5: COMPUTE DETERMINANT
If you select option 5, the computer calculates and displays the determinant of the matrix. Press any key to return to the selection list.

OPTION 6: SOLVE EQUATIONS
If you select option 6, the computer displays a system of equations based on the matrix. The coefficients for the unknowns in the equations are represented by the elements in the matrix entered in option 1. Next, the computer then asks you to enter the value for the equations. It then computes and displays the values for the variables in the equations. Press any key to return to the selection list.

OPTION 7: REDIRECT OUTPUT
If you select option 7, the computer gives you the option of displaying calculated results on the screen or printing them on the Thermal Printer. To display results on the screen, press ENTER. To print the results, type TP and press ENTER. In either case, the computer automatically returns to the selection list.

OPTION 8: EXIT
To return to TI BASIC, select option 8, or press SHIFT C (CLEAR).

Find the inverse of the maxtix:

$$\begin{vmatrix} 2 & 3 \\ 1 & 1 \end{vmatrix}$$

and solve the equations:

```
2X + 3Y =  1
 X + Y  = -1
```

First, select option 1, ENTER MATRIX. Enter the size of the matrix, which is two. Then enter the elements in the rows as shown below:

```
1)    2   3
2)    1   1
```

Press any key to return to the selection list. Then select option 4, INVERT MATRIX, and the computer displays:

```
DETERMINANT = -1

-1      3
 1     -2
```

Now, press any key again, and select option 6, SOLVE EQUATIONS. The computer displays:

```
ENTER EQUATION VALUES:

2A + 3B = ?  (Enter 1)
 A + B  = ?  (Enter -1)
```

The computer then calculates and displays the values for A and B:

```
A = -4
B =  3
```

To return to the selection list, press any key.