No. 147.

THE INSTITUTION OF
POST OFFICE ELECTRICAL ENGINEERS

The Design and Construction of Electric Wave Filters

BY

R. J. HALSEY, B.Sc. (Hons.), A.C.G.I., D.I.C.

A PAPER

Read before the London Centre of the Institution on the 13th December, 1932.

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Introduction.

In order to meet modern demands for the more economical use of transmission lines, the electric wave filter must inevitably find an increasing sphere of usefulness in both telegraphic and telephonic equipment. Although the design of these networks is, and must remain, the work of specialists, it is important that a more widespread understanding of the underlying principles should exist. Several excellent text-books, dealing with filter theory, are available, though these invariably stress the mathematics of the subject. In the present paper an attempt has been made to describe the most important features of the practical designer's art and to show how far the theoretical design may be realised. The practical side of the paper embraces experience at frequencies up to 50 kilocycles per second.

The function of an electric wave filter is to provide a free transmission path for frequencies between certain prescribed limits: in other words, to do, for relatively wide bands of frequencies, what a simple resonant circuit does for narrow bands. Modern circuit conditions call for very sharp cut-offs at the limits of the transmission bands and also for low losses throughout these bands. Filters have been developed along lines which enable the frequency discrimination of two or more sections readily to be added together, and by this means attenuations of 100 decibels or more may be obtained if so desired.

PART I.

General notes on passive four-terminal networks.

Any passive four-terminal network, *i.e.*, one which contains no source of energy, will have electrical properties which, so far as steady state conditions are concerned, can be defined completely by a knowledge of (*a*) the characteristic (image) impedances Z_{01} and Z_{02} at the two ends of the network and (*b*) the propagation constant consisting of a real part (β, the attenuation) and an unreal part ($j\alpha$, the phase shift). If such a network is inserted between a source of energy of internal impedance Z_A and a load Z_B, the effect will be a transmission loss and a phase shift made up of four distinct parts, of which any of the last three may involve negative losses, *i.e.*, gains. The algebraical sum of these component losses must, however, be positive or zero.

(1) Due to the propagation constant, the voltage is attenuated in the ratio $e^{\beta + j\alpha}$.

(2) Due to the input impedance mismatch, there is a reflection loss ratio $\dfrac{2\sqrt{Z_A Z_{01}}}{Z_A + Z_{01}}$ the real part being the true loss and the imaginary part the phase shift in radians.

(3) Due to the output impedance mismatch, there is a reflection loss ratio $\dfrac{2\sqrt{Z_B Z_{02}}}{Z_B + Z_{02}}$

(4) Due to secondary reflections or interaction between the two terminal impedance mismatches, there is an interaction loss ratio

$$1 - \left\{ \frac{Z_A - Z_{01}}{Z_A + Z_{01}} \cdot \frac{Z_B - Z_{02}}{Z_B + Z_{02}} e^{-2(\beta + j\alpha)} \right\}$$

If the generator had been directly connected to the load there would have been a loss ratio $\dfrac{2\sqrt{Z_A Z_B}}{Z_A + Z_B}$, hence the net insertion loss is

$$\beta + \log_e \left| \frac{Z_A + Z_{01}}{2\sqrt{Z_A Z_{01}}} \right| + \log_e \left| \frac{Z_B + Z_{02}}{2\sqrt{Z_B Z_{02}}} \right|$$

$$+ \log_e \left| 1 - \left\{ \frac{Z_A - Z_{01}}{Z_A + Z_{01}} \cdot \frac{Z_B - Z_{02}}{Z_B + Z_{02}} e^{-2(\beta + j\alpha)} \right\} \right|$$

$$- \log_e \left| \frac{Z_A + Z_B}{2\sqrt{Z_A Z_B}} \right| \text{ nepers.}$$

The phase shift in radians will be the sum of the individual phase shifts just discussed.

Recurrent networks.

A recurrent network is one which consists of a plurality of simple networks which are joined together on an image impedance basis and which therefore give rise to no reflections at the junctions. The propagation constant of such a network is precisely the sum of the propagation constants of the component sections and the only reflections to be considered are those at its terminals.

Recurrent networks consist broadly of three types

(*a*) Ladder networks
(*b*) Lattice or bridge networks
(*c*) Bridged T networks

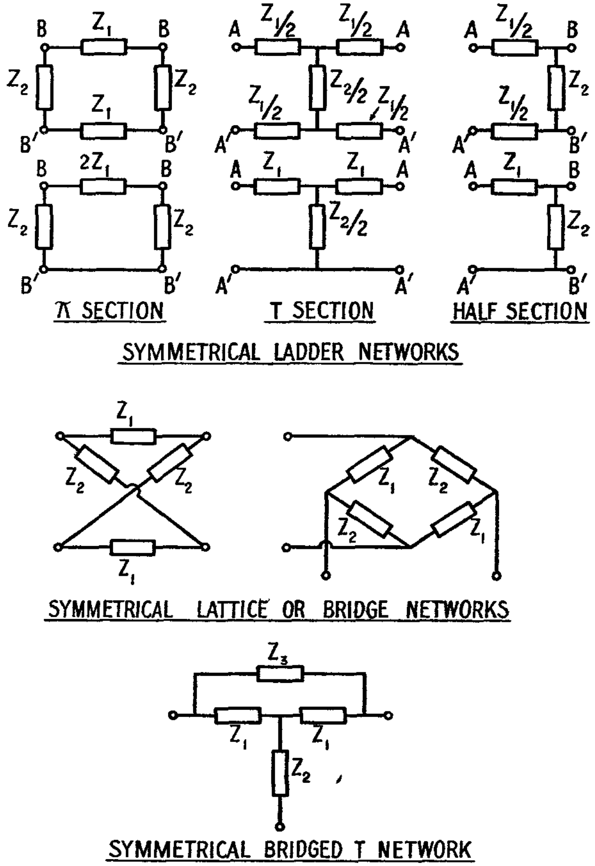
and these are mutually convertible, *i.e.*, for any ladder network there are other equivalent structures. These cannot necessarily be realised in practice, however, owing to inability to produce a reactance which has the properties of a negative capacity. Negative inductances can often be obtained as mutual inductances.

Ladder networks.

Ladder networks (except some involving mutual inductance) can be built up of half sections into the well-known T and π structures and so an examination of the properties of the half section will suffice. The propagation constant of a half section is half that of a whole section, whether of T or π formation. Figure 1 shows the formation of ladder sections in their balanced and unbalanced forms, suitable for insertion in balanced and unbalanced equipments respectively. The unbalanced structures are convenient for design purposes, as they can readily be converted into balanced structures.

A half section network has two image impedances which are generally unequal and which are known as the mid-series and mid-shunt impedances (AA^1 is a mid-series point and BB^1 is a mid-shunt point). Thus a T section has two mid-series impedances and is a mid-series section; a π section has two mid-shunt impedances and is a mid-shunt section.

Electric wave filters form a class which can be conveniently developed as ladder networks and it is in this form that they will now be considered.



RECURRENT NETWORKS.

FIG. 1.

Wave filters.

There are four types of wave filter :—

(1) Low-pass.
(2) High-pass.
(3) Band-pass.
(4) Band-stop.

Complex filters, involving more than one of the above features, may be designed but are seldom required. All the impedance elements employed in filters are primarily reactances.

The properties of a ladder filter section are calculated as follows :—

Image impedance at AA^1, $Z_{01} = \sqrt{Z_1^2 + Z_1Z_2}$

,, ,, ,, BB^1, $Z_{02} = \dfrac{Z_2^2Z_1}{Z_2 + Z_1}$

Propagation constant, $\sinh \dfrac{\theta}{2} = \sqrt{\dfrac{Z_1}{Z_2}}$

where $\dfrac{\theta}{2}$ refers to the half section.

In the absence of any simplified formulæ, it is always possible to calculate direct from these basic equations and this has to be resorted to in some cases. When $\sinh \dfrac{\theta}{2}$ is entirely real or entirely unreal, θ is conveniently determined from tables such as those compiled by the Smithsonian Institute; but, when $\sinh \dfrac{\theta}{2}$ is complex, it is necessary to use a Kennelly Atlas or tables of complex functions. Now it is fundamental to filter theory that $\sinh \dfrac{\theta}{2}$ is never complex if resistance is neglected and so, provided elements with low dissipation are employed, we can reserve considerations of complex quantities to

(*a*) losses in the transmission band,
(*b*) limiting values near attenuation peaks.

Apart from these minor corrections, we have

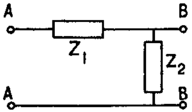
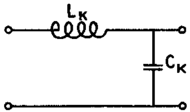
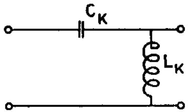
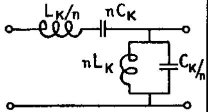
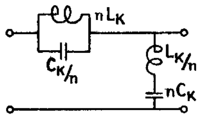
$\sinh \dfrac{\theta}{2} = \sinh \dfrac{\beta}{2}$ in the attenuation bands.

and $\sinh \dfrac{\theta}{2} = \sinh \dfrac{a}{2}$ in the transmission bands.

Basic sections.

In considering the properties of filter sections, we have to examine, first the basic or parent sections, and second, their derivations. The basic half sections of the four types are shown in Fig. 2, together with their image impedances and propagation constants.

The values of L_k and C_k can be readily calculated, for, in the case of the high-pass and low-pass sections,

	GENERAL	LOW PASS	HIGH PASS	BAND PASS	BAND STOP
HALF SECTION					
Z_{01} (at AA)	$\sqrt{Z_1^2 + Z_1 Z_2}$	$\sqrt{1 - x^2}\ R_0$	$\sqrt{1 - \left(\frac{1}{x}\right)^2}\ R_0$	$\sqrt{1 - \left(\frac{1-x^2}{nx}\right)^2}\ R_0$	$\sqrt{1 - \left(\frac{nx}{1-x^2}\right)^2}\ R_0$
Z_{02} (at BB)	$\sqrt{\frac{Z_2^2 Z_1}{Z_1 + Z_2}}$	$\frac{R_0}{\sqrt{1-x^2}}$	$\frac{R_0}{\sqrt{1-\left(\frac{1}{x}\right)^2}}$	$\frac{R_0}{\sqrt{1-\left(\frac{1-x^2}{nx}\right)^2}}$	$\frac{R_0}{\sqrt{1-\left(\frac{nx}{1-x^2}\right)^2}}$
Sinh $\theta/2$	$\sqrt{\frac{Z_1}{Z_2}}$	x	$\frac{1}{x}$	$\frac{1-x^2}{nx}$	$\frac{nx}{1-x^2}$
Cosh θ	$1 + 2\frac{Z_1}{Z_2}$	$1 - 2x^2$	$1 - \frac{2}{x^2}$	$1 - 2\left(\frac{1-x^2}{nx}\right)^2$	$1 - 2\left(\frac{nx}{1-x^2}\right)^2$
L_K		$\frac{R_0}{\omega_0}$	$\frac{R_0}{\omega_0}$	$\frac{R_0}{\omega_m}$	$\frac{R_0}{\omega_m}$
C_K		$\frac{1}{\omega_0 R_0}$	$\frac{1}{\omega_0 R_0}$	$\frac{1}{\omega_m R_0}$	$\frac{1}{\omega_m R_0}$
		where $x = \left(\frac{\omega}{\omega_0}\right)$ " R_0 = nominal characteristic impedance		where ω_1 and ω_2 are the cut off frequencies $\omega_m = \sqrt{\omega_1 \omega_2}$; $x = \frac{\omega}{\omega_m}$; $n = \frac{\omega_2 - \omega_1}{\omega_m}$	

PROPERTIES OF BASIC FILTER SECTIONS.

FIG. 2.

$$\omega_0 L_k = \frac{1}{\omega_0 C_k} = R_0$$

where R_0 is the nominal characteristic impedance (usually the closing impedance) and ω_0 is the cut-off frequency in radians per second.

In the case of the band filters

$$\omega_m L_k = \frac{1}{\omega_m C_k} = R_0$$

where R_0 is as before and ω_m is the logarithmic mean frequency of the band. The constant n is the ratio

$$\frac{\text{band width}}{\text{mean band frequency}} = \frac{f_2 - f_1}{\sqrt{f_1 f_2}}$$

When terminated by their image impedances the above half sections have the following important properties:—

(1) Over certain precise frequency bands the propagation constant is either wholly real or wholly imaginary. This is the true criterion of the transmission bands.

(2) Throughout the transmission bands the image impedances are non-reactive, one rising to infinity and one falling to zero at the cut-off frequencies.

(3) The product of the two image impedances is constant at all frequencies ($= R_0^2$).

(4) The only frequencies of infinite attenuation are zero and infinity (ω_m in the case of the band stop section).

The first two properties are common to all true filter sections, but the others are peculiar to the basic or " constant K " structures.

Although of considerable value, the basic half sections have two disadvantages which restrict their use.

(1) Their image impedances vary considerably over the important part of the transmission bands and when working between fixed resistances (which is approximately the normal working condition) considerable reflection losses occur.

(2) The points of infinite attenuation are not economically located.

Derived sections.

The derived type of half section is designed to overcome these disadvantages. It has two unequal image impedances, one of which is identical with that of the basic half section, to which it can obviously be joined without reflections. The other image impedance, which can be conveniently presented to the closing resistance, may be made substantially constant over the greater part of the transmission band. Moreover, the attenuation now becomes infinite at some finite frequency. In the case of the band-pass filter, the two attenuation peaks may be adjusted separately by a further refinement.

The derived type of section has therefore two distinct uses:

(*a*) Either a whole section or a half section may be used to give attenuation peaks at finite frequencies in the attenuation bands.

(*b*) A terminal half section may be used to present a flat impedance to a fixed resistance load.

The calculation of derived sections involves the use of a derivation parameter m, which is always less than unity except in a few uncommon mutual inductance sections. The properties of the simplest derived sections are shown in Figs. 3 and 4.

Low-pass and high-pass filters.

The properties of low-pass and high-pass filters have been fully described in an earlier paper read before this Institution* and no further theoretical discussion will be given. The properties are indicated in Figs. 2, 3 and 4.

Band-pass filters.

Band-pass filter sections comprise more variations than any of the other types, and the degree of flexibility is such that the design information cannot be given completely in graphical form. For the inexperienced designer this constitutes a considerable handicap as the properties of each section have to be calculated instead of being read from a graph. The necessary calculations are rather laborious and unless one has previous experience as a guide, much work is likely to be wasted.

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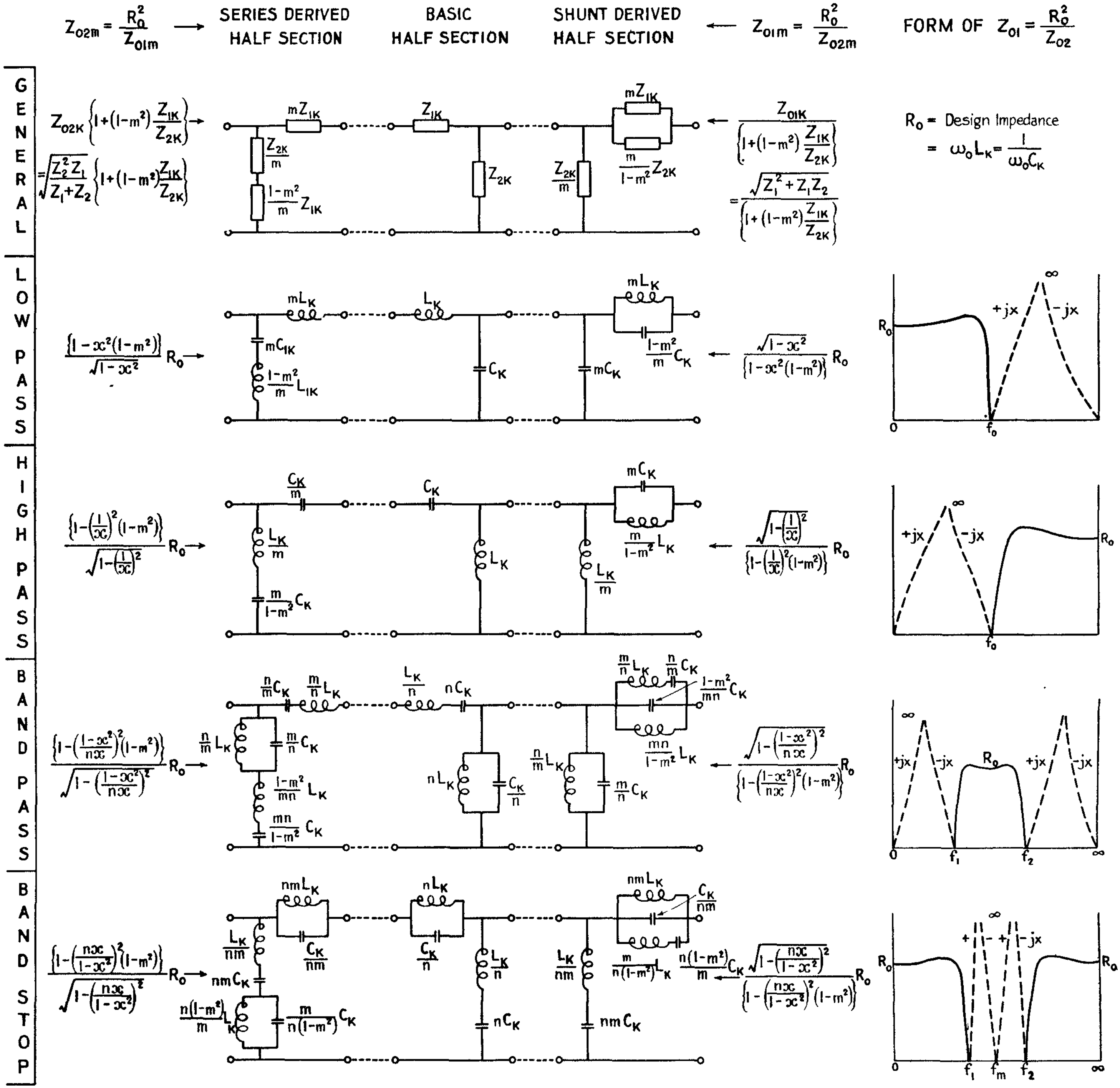
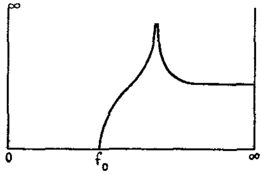
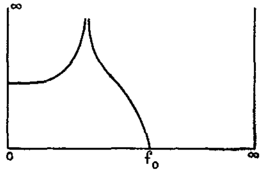
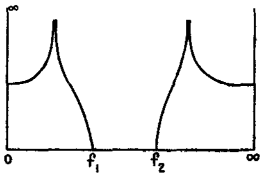
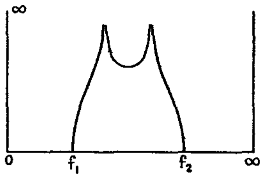
* I.P.O.E.E. Paper No. 143, "Electric Wave Filters," by G. J. S. Little, read December, 1931.



DERIVED HALF SECTIONS AND THEIR IMPEDANCES.
FIG. 3.

TYPE OF SECTION	Sinh $\theta/2$	FORM OF ATTENUATION CURVE
GENERAL	$\sqrt{\frac{Z_1}{Z_2}}\left\{\frac{m}{1+(1-m^2)\frac{Z_1}{Z_2}}\right\}$	
LOW PASS	$\frac{mx}{1-(1-m^2)x^2}$ where $x=\frac{f}{f_0}$	
HIGH PASS	$\frac{mx}{x^2-(1-m^2)}$ where $x=\frac{f}{f_0}$	
BAND PASS	$\frac{m(1-x^2)}{\sqrt{(1-m^2)(1-x^2)^2-n^2x^2}}$ where $x=\frac{f}{\sqrt{f_1f_2}}$	
BAND STOP	$\frac{mnx}{\sqrt{(1-m^2)n^2x^2-(1-x^2)^2}}$ where $x=\frac{f}{\sqrt{f_1f_2}}$	

ATTENUATION OF DERIVED FILTER SECTIONS.

FIG. 4.

In the first place, corresponding to each high or low-pass basic section, there will be a family of band-pass sections, with various ratios of band width to mean frequency (n). These are shown in Figs. 5 and 6.

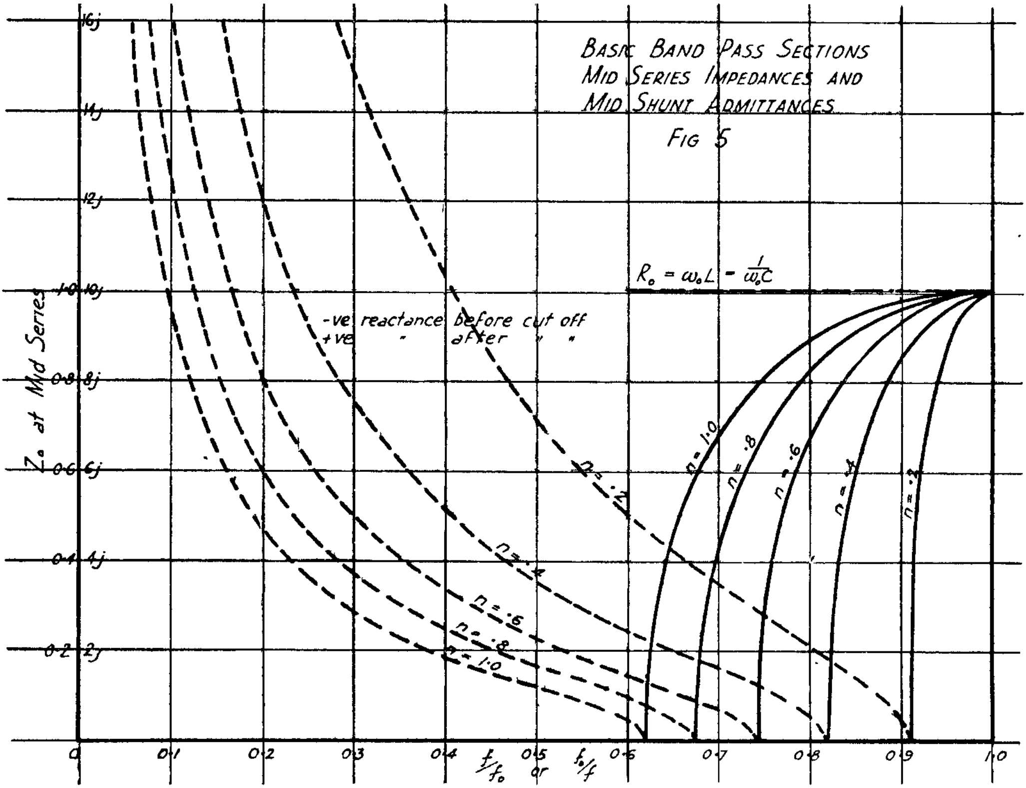


FIG. 5.

The derivation of filter sections in terms of a single parameter *m* has been discussed. This method is, of course, applicable to band-pass filters and gives a series of sections whose properties are given in Figs. 7 and 8 for a value of $n = 0.316$. It will be observed that in these sections, the frequencies of infinite attenuation are symmetrically disposed

ATTENUATION IN NEPERS.

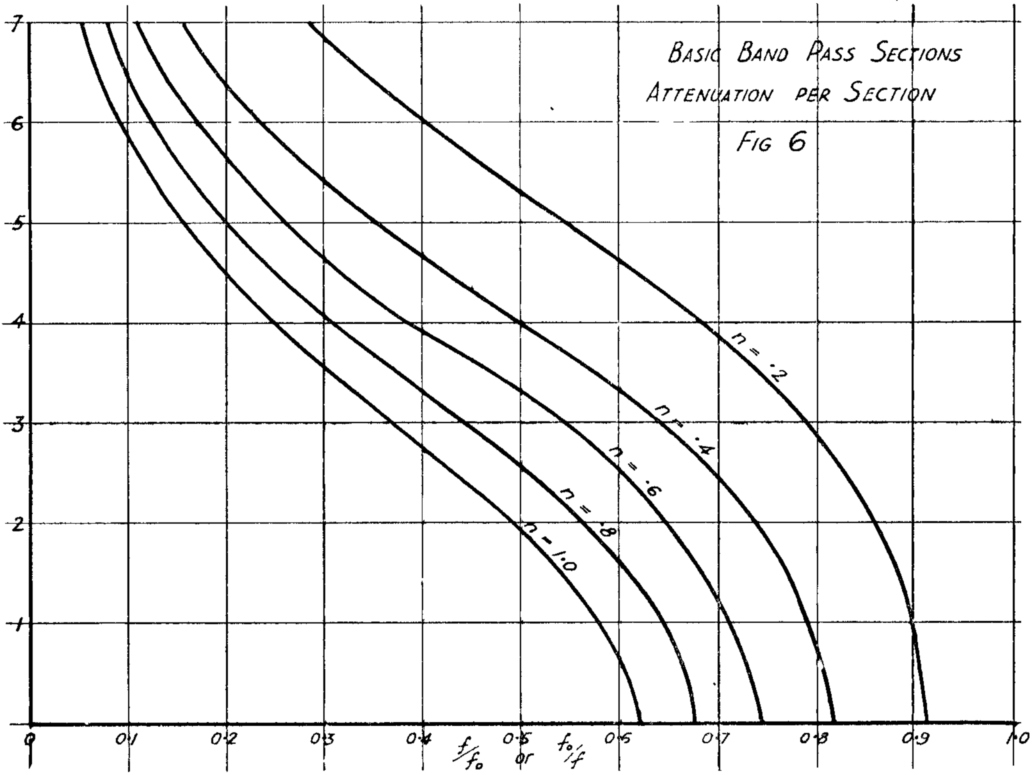


FIG. 6.

(logarithmically) about f_m. They are known as symmetrical six-element sections.

A further degree of flexibility is obtained if we derive a band-pass section in terms of a parameter which varies with

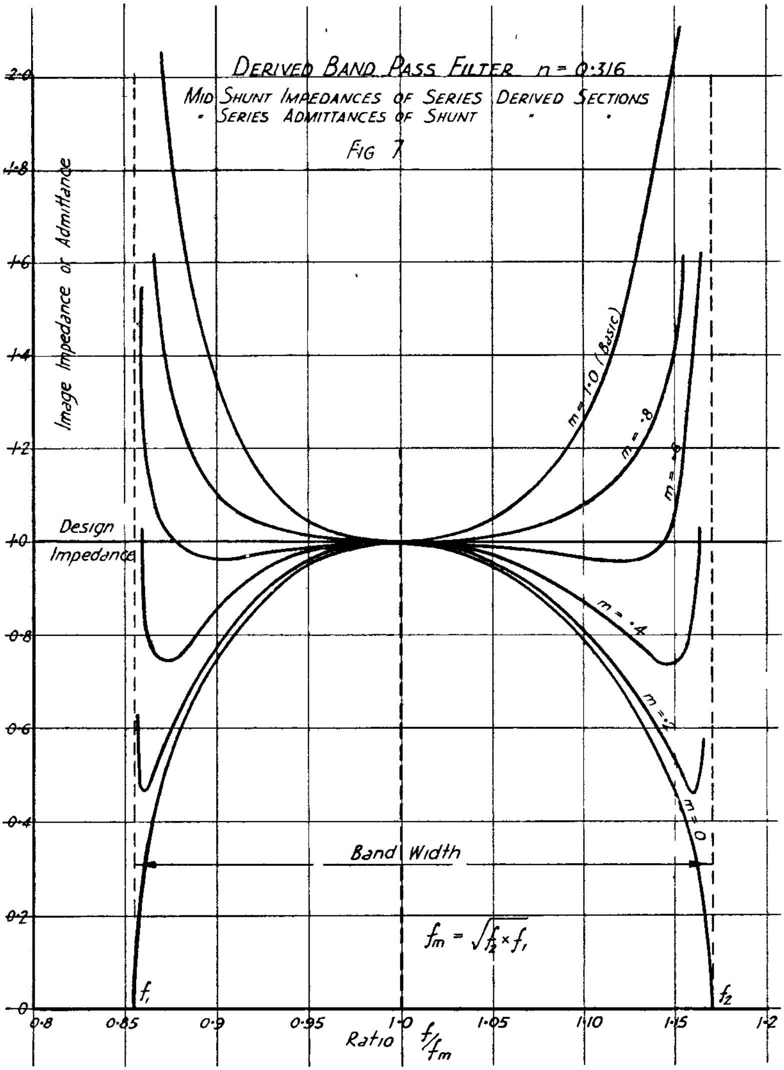


FIG. 7.

frequency. This is equivalent to using two parameters m_1 and m_2, such that m_1 applies to the derived inductance and m_2 applies to the derived capacity. This gives a series of sections, substantially similar to the single derived sections, but with the two attenuation peaks independently adjustable. Such sections are known as dissymmetrical six-element

DERIVED BAND PASS FILTER $n=0.316$.
ATTENUATION PER SECTION.

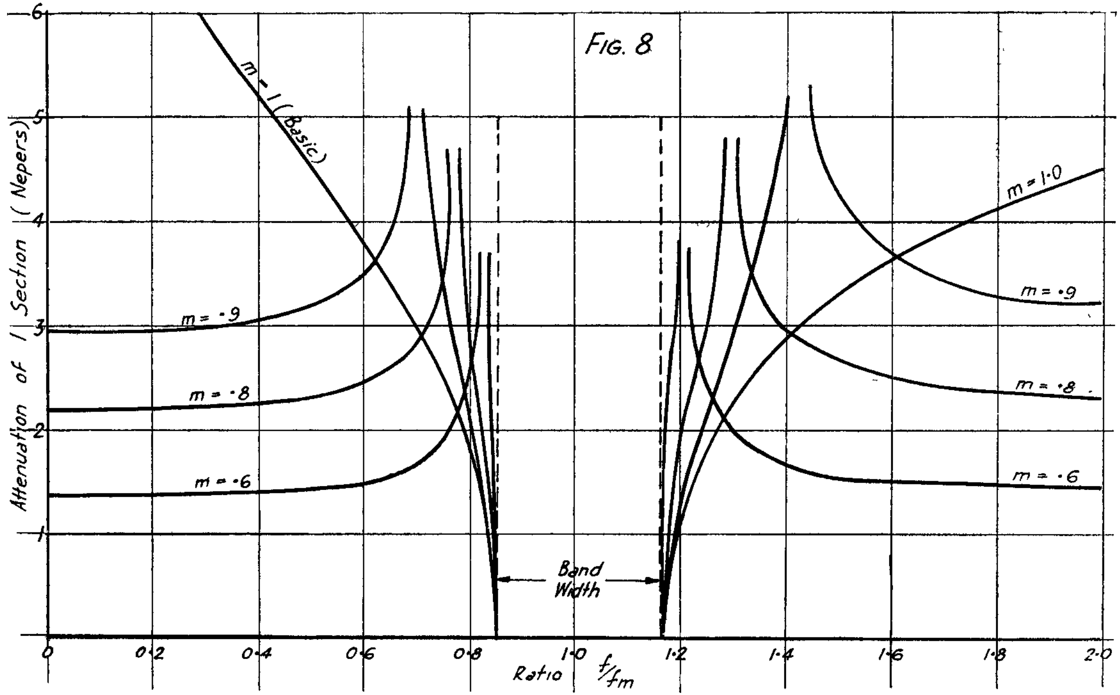


FIG. 8.

sections and form the most generalised type of band-pass section.

By modifying the dissymmetrical six-element section so that one attenuation peak is at zero or infinite frequency, one element disappears and a five-element section is formed.

By making one attenuation peak of a dissymmetrical six-element section coincide with one or other of the cut-off frequencies, two elements disappear and a four-element section is formed.

By combining the two simplifications above, three elements disappear, resulting in the formation of a three-element section.

The general properties of these simplified double-derived sections are shown in Fig. 9.

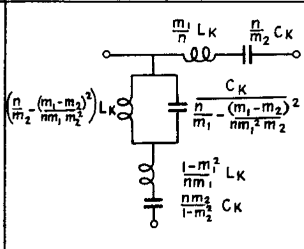
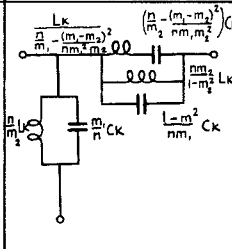
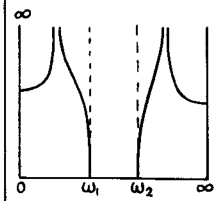
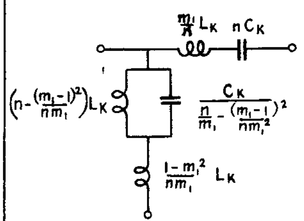
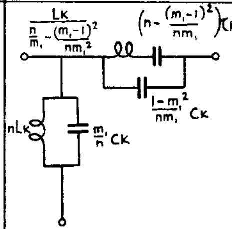
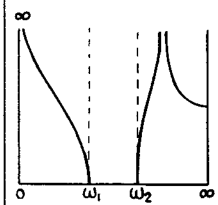
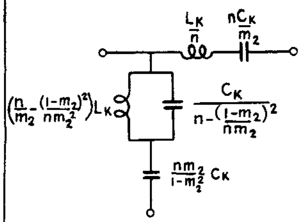
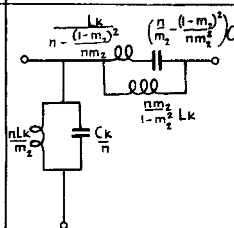
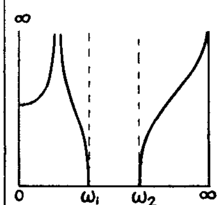
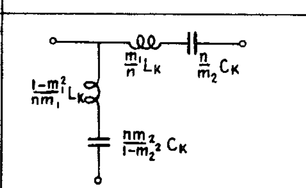
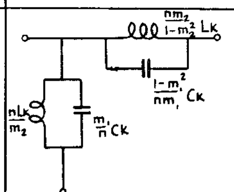
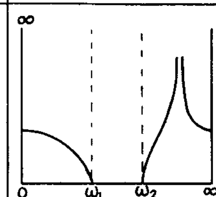
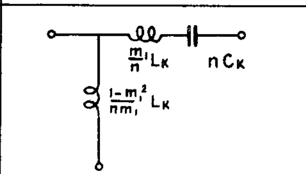
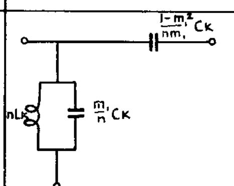
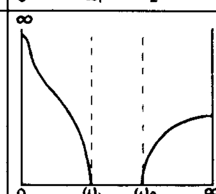
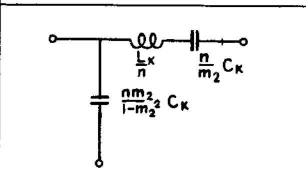
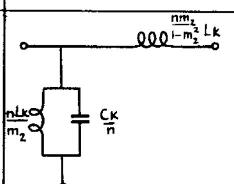
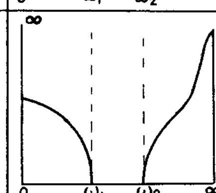
Design information.

It will be understood that, provided care is taken to connect together the half sections on an image impedance basis, we need only be concerned with the impedances of the end sections, whereas the propagation constants of all sections are equally important. This is fortunate from the point of view of presentation of design data as all the image impedances need not be discussed. The complete list of band-pass types consists of:—

1. Basic or " constant K " type.
2. Six-element derived type.
3. Five-element derived type.
   a. Infinite attenuation at zero frequency.
   b. Infinite attenuation at infinite frequency.
4. Four-element derived type.
   a. Infinite attenuation below the band.
   b. Infinite attenuation above the band.
5. Three-element derived type.
   a. Infinite attenuation at zero frequency.
   b. Infinite attenuation at infinite frequency.

Each of the above types, with the exception of the basic type, is available either as a mid-shunt or a mid-series derivation. The data can be narrowed down, however, by the acceptance of the following facts:—

1. Only a six-element network is likely to be used for terminating.
2. The four-element network, with infinite attenuation below the band, may be taken as the standard to which all others may be referred.
3. The impedance of a mid-series derived section is the admittance of the corresponding mid-shunt

ELEMENTS	SERIES DERIVED HALF SECTION	SHUNT DERIVED HALF SECTION	CONDITIONS	FORM OF ATTENUATION CURVE	Sinh $\theta/2$ ($\theta/2$ REFERS TO ½ SECTION)	FREQUENCIES OF PEAK ATTENUATION
6			m_1 & m_2 are independent but $\frac{\omega_2}{\omega_1} > \frac{m_1}{m_2} > \frac{\omega_1}{\omega_2}$		$\frac{m_2 - m_1 x^2}{\sqrt{x^4(1-m_1^2) - x^2(n^2+2m_1m_2) + (1-m_2^2)}}$	$x_1^2 + x_2^2 = \frac{n^2+2-2m_1m_2}{1-m_1^2}$ $x_1^2 x_2^2 = \frac{1-m_2^2}{1-m_1^2}$
5			$m_2 = 1$		$\frac{1 - m_1 x^2}{\sqrt{x^4(1-m_1^2) - x^2(n^2+2-2m_1)}}$	$x_1 = 0$ $x_2 = \sqrt{\frac{n^2+2-2m_1}{1=m_1^2}}$
5			$m_1 = 1$		$\frac{m_2 - x^2}{\sqrt{(1-m_1^2) - x^2(n^2+2-2m_2)}}$	$x_2 = \infty$ $x_1 = \sqrt{\frac{1-m_1^2}{n^2+2-2m_2}}$
4			$\frac{m_1}{m_2} = \frac{\omega_2}{\omega_1}$ or $\frac{m_1}{m_2} = \frac{\omega_1}{\omega_2}$		$\frac{m_1\sqrt{x^2 - \frac{\omega_1}{\omega_2}}}{\sqrt{x^2(1-m_1^2) - (\frac{\omega_2}{\omega_1} - \frac{\omega_1}{\omega_2} m_1^2)}}$ or $\frac{m_1\sqrt{x^2 - \frac{\omega_1}{\omega_2}}}{\sqrt{x^2(1-m_1^2) - (\frac{\omega_1}{\omega_2} - \frac{\omega_2}{\omega_1} m_1^2)}}$	$x = \sqrt{\frac{\frac{\omega_2}{\omega_1} - \frac{\omega_1}{\omega_2} m_1^2}{1-m_1^2}}$ or $x = \sqrt{\frac{\frac{\omega_1}{\omega_2} - \frac{\omega_2}{\omega_1} m_1^2}{1-m_1^2}}$
3			$m_2 = 1$ and $m_1 = \frac{\omega_1}{\omega_2}$		$\sqrt{\frac{\frac{\omega_2}{\omega_1} - x^2}{x^2\left\{\left(\frac{\omega_2}{\omega_1}\right)^2 - 1\right\}}}$	$x = 0$
3			$m_1 = 1$ and $m_2 = \frac{\omega_1}{\omega_2}$		$\sqrt{\frac{1 - \frac{\omega_2}{\omega_1} x^2}{\left(\frac{\omega_2}{\omega_1}\right)^2 - 1}}$	$x = \infty$

Dissymmetrical Band Pass Half Sections Attenuation Characteristics.

Fig. 9.

derived section. The two bear inverse relationship to the design impedance R_0.

The fact that a four-element network may be used as a standard depends on the equivalence of certain propagation constants:

1. A six-element section has a propagation constant equal to that of two four-element networks with corresponding critical frequencies.
2. A three-element network is a special case of a four-element network in which the frequency of infinite attenuation is either zero or infinity.
3. A five-element network has a propagation constant equal to that of one four-element network plus one three-element network.
4. A basic section has a propagation constant equal to the sum of the propagation constants of two three-element networks.

These identities are true if the dissipation constants (*i.e.*, resistance/reactance) of coils and condensers are the same in the equivalent networks.

In addition to the above relationships, if we have a curve of the propagation constant for a four-element section with a peak below the band, then by substituting $\frac{\omega_0}{\omega}$ for $\frac{\omega}{\omega_0}$ we have the corresponding curve for a four-element section with a peak above the band. This will be equivalent to interchanging the values of the parameters m_1 and m_2.

For design purposes, therefore, it is possible to build up a filter structure entirely in terms of three- and four-element networks, the necessary terminating six-element networks being included as equivalent four-element networks.

Having obtained the desired attenuation characteristics in this way, the sections can be grouped into convenient five- and six-element structures. This grouping is desirable as it saves one inductance and one condenser per section, and in some cases more convenient reactance values may be employed. Suitable six-element half sections will have been included for terminating the filter.

Note on three-element band-pass sections.

The general case of a filter with a series resonant circuit in the series arm and a parallel resonant circuit in the shunt arm is shown in Fig. 10.

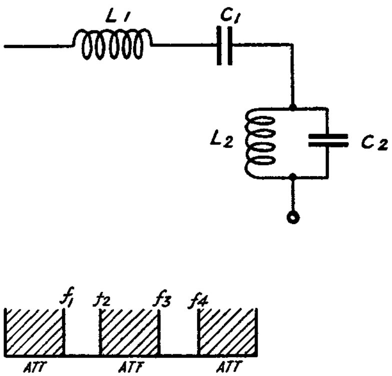


FIG. 10.

Such a section will possess, in general, two transmission bands and three attenuating bands and these bands may be located as required. If a single band only is desired, three alternatives are available:

1. Make $f_2 = f_3$, thus giving two confluent bands. This is equivalent to making $L_1C_1 = L_2C_2$ and gives the " constant K " band pass filter.

2 and 3. Make $f_1 = f_2$ or $f_3 = f_4$, thus giving a single band. This is equivalent to eliminating one of the four elements and gives one or other of the three-element sections discussed elsewhere.

This clearly emphasises the fundamental difference between a " constant K " and a three-element section.

Band-stop filters.

Band-stop filters form a comparatively unimportant class and their variety is the same as that of low-pass and high-pass filters. In other words the band attenuation must be symmetrical (logarithmically) about the mean band frequency. The only available sections are the basic network and its mid-series and mid-shunt derivations with a single parameter m. The properties of typical band-stop sections are shown in Figs. 2, 3 and 4.

The formation of band filters by means of high-pass and low-pass sections.

It will be clear that it is possible to construct an approximation to a band-pass filter by means of high-pass and low-pass sections in series. Similarly a band-stop filter can be formed with high-pass and low-pass sections in parallel. This has certain advantages and disadvantages which must be carefully considered :

Advantages.

(1) Design is easier and one side of the band may be modified without reference to the other side of the band.

(2) In certain cases, the dissipation losses in the transmitting band may be less. This is due to the fact that, in complex arms, reactances may tend to cancel out, whereas resistance losses are always additive; this increases the dissipation constant for the arm.

(3) The higher dissipation constant discussed in (2) will limit the attenuation peaks and, for given coil angles, the attenuation peaks are always blunter in band-pass filters than in high-pass and low-pass filters.

Disadvantages.

(1) Even assuming that care is taken to match terminal impedances as well as possible over the transmitting band, there will be greater terminal mismatch than in the case of true band filters.

(2) There will be a junction mismatch between the high-pass and low-pass sections (in the band-pass case) which is entirely absent in the case of the true band-pass filter.

(3) For a given performance, a band-pass filter is more economical of elements : this does not apply to the attenuation peaks, however.

As a general rule it may be taken that for low values of $\frac{\text{bandwidth}}{\text{mean frequency}}$ $(= n)$ it is desirable to use band-pass sections, but for relatively large band widths high- and low-pass filters may give a better performance though slightly less economical of filter elements.

Filter design requirements.

The requirements of a filter can usually be set out under the following headings:—

1. Impedance in the transmission band. Since the filter will presumably work between substantially constant non-reactive impedances, it will be desired to maintain a reasonably constant terminal impedance in order to avoid reflections and their associated losses. With a single derivation it is possible to maintain a constant impedance within ±5% over about 90% of the band width (this varies somewhat with various filters) or within ±10% over about 95% of the band width.

   Flat terminal impedances may also be important on the score of cross-talk in a transmission system, for reflections give rise to a type of cross-talk which is difficult to eliminate with line transpositions. The requirements for this purpose may be extremely severe in certain cases, but it may be regarded as a secondary reason for striving after a close terminal match.

2. Impedance in the attenuating range. Generally speaking, we have the choice of a low impedance or a high impedance at certain frequencies in the attenuating bands, by using either mid-series or mid-shunt derivations. (The derived impedances of the two types bear inverse relationship to the design impedance R_0). In some cases it may be important which of these alternatives is chosen.

3. Losses in the transmitting band. These consist of two distinct parts: (*a*) reflection losses due to terminal mismatch, referred to in (1) above, which give rise to a rolling attenuation throughout the band; (*b*) losses due to dissipation in the resistance of the filter elements; these losses increase towards the cut-off frequencies and, if it is desired to work near these frequencies, it is necessary that low dissipation coils shall be used.

4. Minimum attenuation in the attenuating bands. From consideration of the problem in hand it will be possible to determine the minimum attenuation to be obtained at various frequencies. In the case

of a transmission system, this will usually involve fixing a minimum signal-to-crosstalk ratio to be aimed at and making due allowance for the relative levels of the interfering frequencies. Allowance may also be made in some cases for the frequency sensitivity of the human ear. In many cases it will be desirable to have considerably higher attenuations at pre-determined frequencies and these should be noted. The resultant attenuation curve will consist of several peaks and troughs, so that it is usually economical to arrange the peaks to occur at important interfering frequencies.

5. Phase shift in the tranmission bands. It is sometimes desirable to design filters in which the phase shift is as low as possible, at the important transmission frequencies. This involves the use of derived sections as far as possible.

6. Propagation time in the transmission bands. Where a number of filters are fitted in a transmission system, the time of propagation may become important. In the first place, it may be desirable to minimise the difference of propagation times of the transmitted frequencies, in order to obviate or facilitate phase compensation. Secondly, it may be important to minimise the compensated propagation time. Either of these considerations suggests the use of derived sections, as far as possible; the use of a minimum number of sections; and the avoidance of working near the limiting frequencies of the band.

From the foregoing it will be appreciated that there can be no one ideal design to suit given requirements, but that a suitable result can be achieved with a minimum number of components by a careful and painstaking investigation of possible arrangements. In the case of high-pass and low-pass sections the design information is readily presented in the form of curves, but in the case of band filters, with their greater flexibility, this is not possible and typical curves only can be given.

Effect of terminal reflections.

As indicated in the section on four-terminal networks, the propagation constant of a reactance structure is only one factor (though desirably the most important one) contributing to the total insertion loss. We have also to concern ourselves with the effect of terminal mismatch. In general there will be a loss at each pair of terminals and, if the attenuation constant is low, this will be modified into a series of peaks and troughs by the interaction factor—the measure of repeated reflections between the terminals. The method of calculating these losses has been given, but a few notes on their importance in wave filters are tabulated below.

1. Interaction factor may be neglected completely in the attenuating bands.
2. In a well designed network, all terminal losses may be neglected in the transmission band except very close to the cut-off frequencies.
3. The effect of terminal mismatch in the attenuating band is, in the usual case of filters working between fixed resistances, to give a reflection gain of about 3 decibels at each terminal. This applies over the whole of the attenuating band except very close to cut-off and at the attenuation peaks of the terminating sections. At these peak frequencies there is a considerable terminal loss, depending on the dissipation constant of the resonant arm.
4. In the case of a high-pass and low-pass filter joined together with a proper impedance correcting network (such as carrier telephone line filters), the net terminal loss will be approximately zero, except at the peak attenuation frequency of the terminal section remote from the junction.

Effect of resistance in the filter elements.

While the effect of resistance can always be dealt with under the heading of propagation constant, the dissipation constants are usually so low that this is not worth while. The propagation constant is usually calculated on the basis of pure reactance arms and the resistance losses are made the subject of an independent calculation where necessary.

Provided that the dissipation constants are low we need only consider resistance losses—

(*a*) In the transmission band;

(*b*) At the peak attenuation frequencies of the individual sections. In the case of the terminating half sections this is doubly important as it affects the attenuation constant proper and also the terminal reflection loss.

Equivalent networks.

Having calculated the reactance arms of a filter network, it is desirable to see if some equivalent network is available with more convenient inductance and capacity values. In particular, large capacities are undesirable on account of their bulk and cost, and large inductances cannot be made to operate at high frequencies. Reactance arms containing two elements only have no alternative forms, but with three elements or more there are equivalent networks whose number increases with the complexity of the arm.

It will be understood that an irreducible reactance arm is one which contains the minimum number of elements and therefore it is impossible to simulate this arm with fewer elements. The criterion for an irreducible arm will be that the number of capacities does not differ from the number of inductances by more than one and also that the number of finite resonant frequencies (including series and parallel resonances) shall be one less than the number of reactance elements.

If we consider, say, four-element reactance arms, then we can draw eight irreducible structures containing this number of elements. Of these, four are potentially equivalent, *i.e.*, they may be made equivalent by suitable choice of values, and the other four are potentially inverse. Two networks are said to bear inverse relationship to an impedance Z_0 when the product of their impedances is Z_0^2 at all frequencies.

The choice of a suitable form of reactance arm will in no way alter the calculated properties of a network, provided that coils and condensers with similar dissipation constants are used in the various configurations of the arm. Of the equivalent networks shown in Fig. 11, those which are usually most convenient are D and H for the four-element arms, and B and D for the three-element arms. A factor which may sometimes decide the type of arm to be used is that, where the network involves a parallel resonant circuit, the self capacity of the coil is readily allowed for.

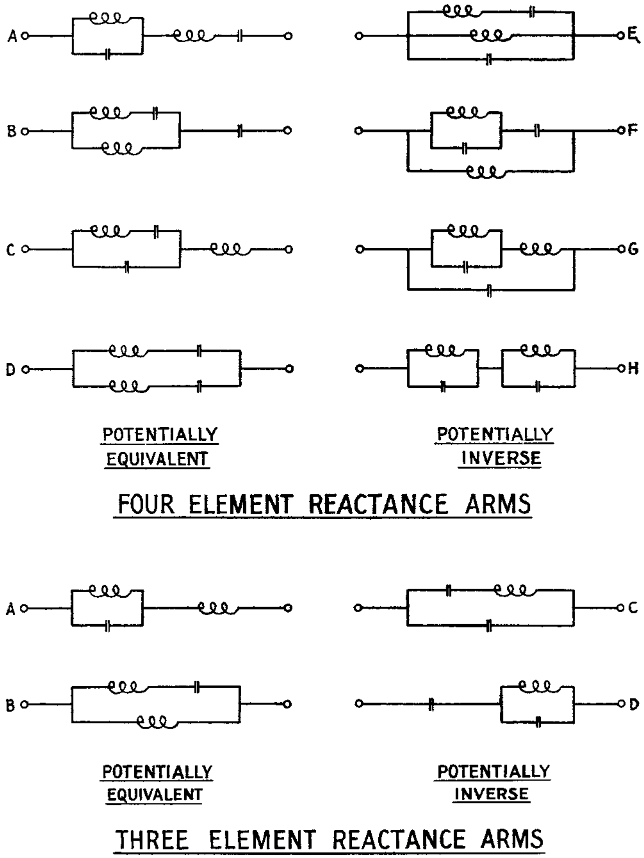


Fig. 11.

Extension of the principle of derived sections.

In all the cases considered, the basic section, from which derived sections have been formed and whose image impedance they share has been of the " constant K " type. " Constant K " means that the series and shunt arms of the basic type are inverse networks. It will be clear that derived sections could also be formed with reference to another section which is itself derived. Such a section is known as a twice-derived section and is employed in rare cases. When such a section is used, its object is to provide a flatter terminal impedance than is possible with an ordinary derived section.

By suitable choice of the derivation parameters m^1 m^{11}, an extremely flat terminal impedance is obtainable. According to Zobel* a section having $m^1 = 0.723$, $m^{11} = 0.4134$ has an impedance which does not vary by more than 2% from the nominal value up to 95% of the cut-off frequency. The propagation constant of such a section is identical with that of a single derived section in which $m = m^1m^{11}$. It should be clear that a twice-derived section cannot be introduced into a normal filter network without reflections, except through the medium of a half section of its immediate parent type. This is illustrated in Fig. 12, in which the series and shunt derivations are shown on opposite sides of the basic half sections.

Bridge filters.

By using the transformations of Fig. 13, it will be possible to obtain equivalent bridge structures for any of the ladder structures already described. These structures will, however, always be more wasteful of elements for the same performance, chiefly owing to the necessity for a balanced filter in the bridge arrangement. This difficulty can be overcome by the use of a differential transformer as in Fig. 12, which shows equivalent networks. The new type of filter is known as a differential bridge filter and its performance is identical with that of a true bridge filter, assuming an ideal transformer. In practice, the effect of the transformer will be of secondary importance. This type of filter section can, of course, be used as a recurrent structure in precisely the same manner as the ladder sections, but it is more convenient to obtain the required characteristics with a single section of greater complexity. Considerable work has been done in this connection in Germany† where bridge filter sections are favoured, largely in view of patent considerations.

Using these complex differential structures, filters can be constructed with about the same economy of elements as in the case of recurrent ladder structures. The design appears to be more complicated, however, and it is doubtful whether they will ever take a prominent position in the equipment manufactured for the Post Office.

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* "Extensions to the Theory and Design of Electric Wave Filters." O. J. Zobel. B.S.T.J., April, 1931.

† E.N.T., July, 1932. "Multiple Bridge Filters," by A. Jaumann.

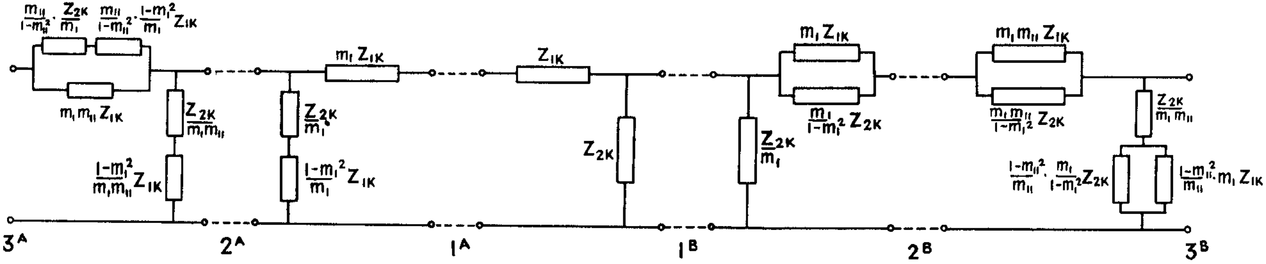


IMAGE IMPEDANCE AT $1^A = Z_{01K} = \sqrt{Z_{1K}^2 + Z_{1K} Z_{2K}}$

" " " $2^A = Z_{02K}\left[1+(1-m_1^2)\frac{Z_{1K}}{Z_{2K}}\right]$

" " " $3^A = \dfrac{Z_{01K}\left[1+(1-m_1^2)\frac{Z_{1K}}{Z_{2K}}\right]}{\left[1+(1-m_1^2 m_{11}^2)\frac{Z_{1K}}{Z_{2K}}\right]}$

IMAGE IMPEDANCE AT $1^B = Z_{02K} = \sqrt{\dfrac{Z_{2K}^2 Z_{1K}}{Z_{1K}+Z_{2K}}}$

" " " $2^B = \dfrac{Z_{01K}}{\left[1+(1-m_1^2)\frac{Z_{1K}}{Z_{2K}}\right]}$

" " " $3^B = \dfrac{Z_{02K}\left[1+(1-m_1^2 m_{11}^2)\frac{Z_{1K}}{Z_{2K}}\right]}{\left[1+(1-m_1^2)\frac{Z_{1K}}{Z_{2K}}\right]}$

PRINCIPLE OF MULTIPLE DERIVATION.

FIG. 12.

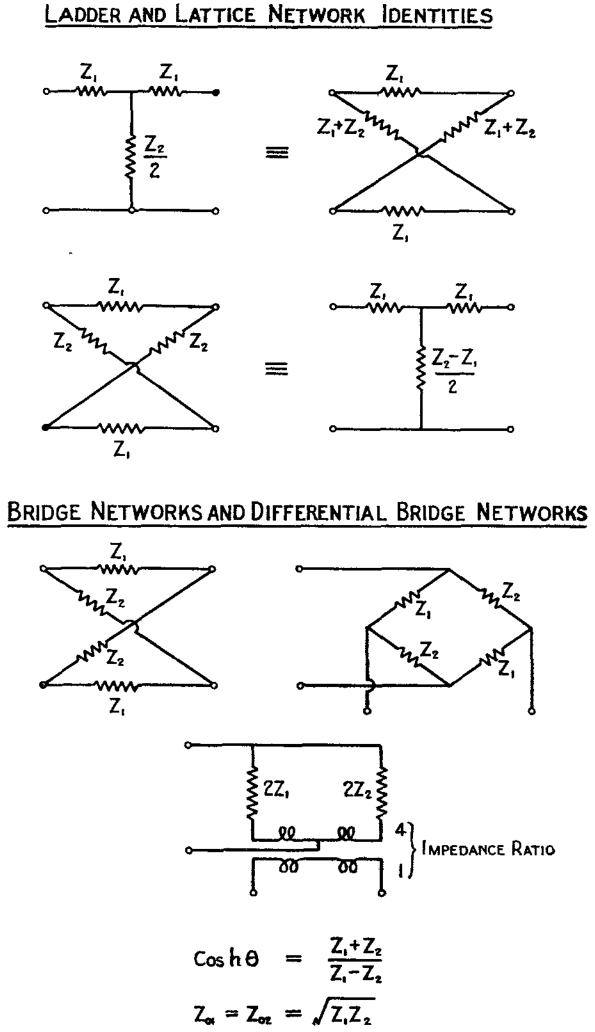


FIG. 13.

Balanced filters.

When a filter is required to work between balanced circuits one of two methods may be adopted :

1. It may be built up as an unbalanced network and terminated by balanced and screened transformers whose winding impedances are high,

compared with the circuit impedance, over the transmitting band.

2. A balanced filter may be used, with the series impedances equally divided between the two legs. This will involve doubling the number of series condensers and also doubling their capacities, but the two halves of each series inductance may be wound on the same core, to give the same total inductance. For normal purposes it is usual to specify a maximum unbalance of 1/200 of the total, between the elements in the two arms, but where phantom circuits are superposed on the filter it may be necessary to have a closer balance. For low-pass filters carrying phantoms an inductance unbalance not exceeding 1/2000 has been specified. In this case also, a low leakage inductance is desirable and for this reason the coils are usually overwound like loading coils.

Mutual inductance in filter networks.

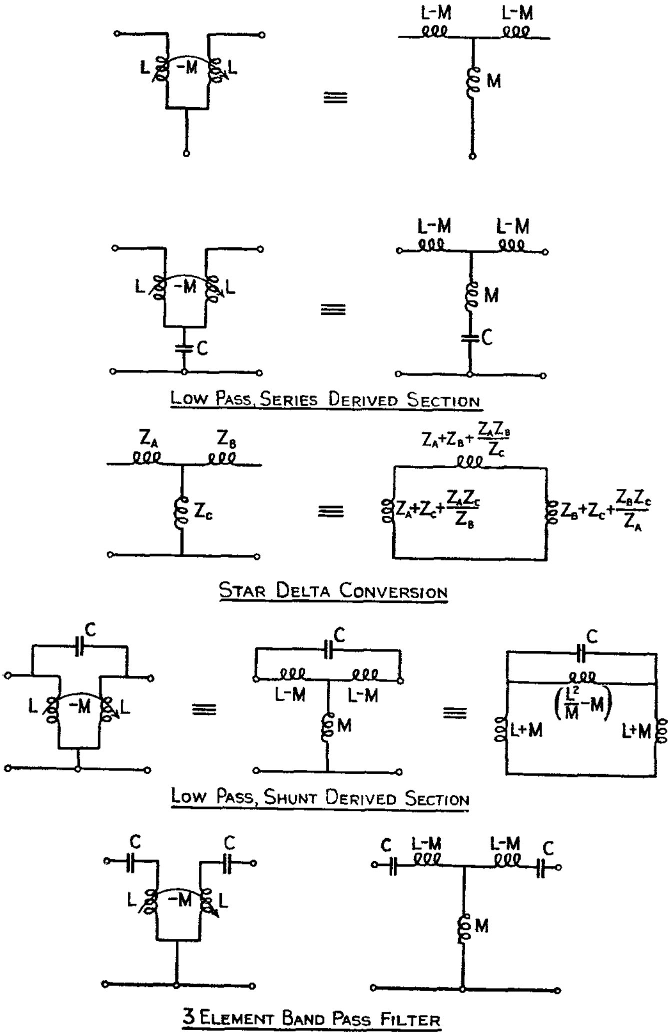
Apart from the use of mutual inductance to obtain balanced networks, it may sometimes be used to economise in the number of elements. Its use depends on circuit identities which are shown in Fig. 14, and which are self-explanatory.

Terminal impedance match at the junction of two filters, teed together on a common line.

Wherever two frequency bands are connected to a common line, it is necessary to ensure that each filter is correctly terminated in its transmitting band. There are two distinct methods of doing this:

1. Where a high-pass and low-pass pair, or a band-pass and band-stop pair are teed together, it is convenient to terminate each with a section having a normal (*i.e.*, constant K) mid-series impedance. If, then, the impedances of the terminal series arms are increased by some fraction (about 0.5, but depending somewhat on the overlap), the two filters can be made to match the line very closely. The match is often better than that of a single filter terminated with a derived section. This is shown, for a high-pass and low-pass pair, in Fig. 15.

2. Where two or more band filters are teed together,



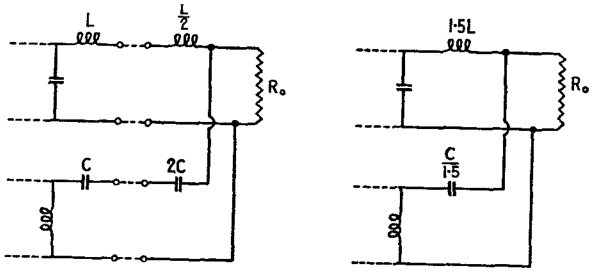
MUTUAL INDUCTANCE IN FILTERS.

FIG. 14.

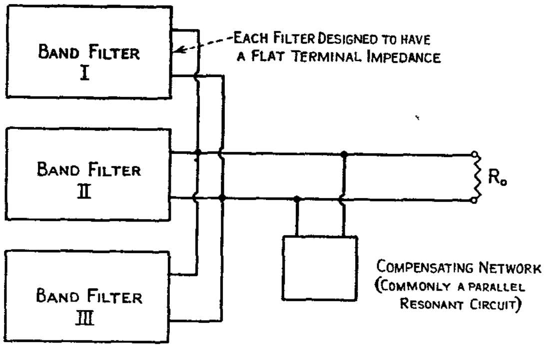
the above arrangement does not give a terminal match. In this case, it is convenient to design a network which, in conjunction with the other one or more band filters, gives a high shunt impedance over the transmitting bands of the various filters. If each of the band filters is then designed to have a flat image impedance at its line terminals, the reflection losses can be made small.

Connecting a high-pass filter to a balanced line.

Where a high-pass filter is connected between a balanced line and unbalanced equipment, this can be done by using an unbalanced structure with a transformer at the line end. With such an arrangement, the impedance of the transformer windings must be high and trouble may occur due to leakage inductance. A device for avoiding this trouble is shown in Fig. 16. A change of impedance may be obtained, if so desired.



METHOD OF TEEING TOGETHER A HIGH PASS AND LOW PASS FILTER PAIR



METHOD OF TEEING TOGETHER BAND PASS FILTERS

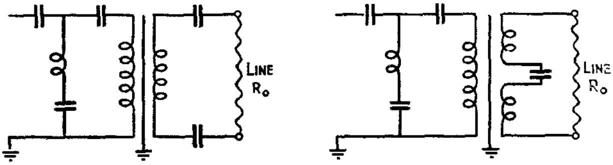
FIG. 15.

When a high-pass and low-pass filter are teed together (as in the case of carrier line filters), the terminating section is of the basic type in each case. This lends itself conveniently to the construction shown. The inductance of the transformer winding is used as a filter element, thus saving a coil and avoiding troubles due to leakage inductance. Where the terminating section is a derived type, the transformer element may be introduced in the second section. A similar construction is also possible in certain types of band-pass filters.

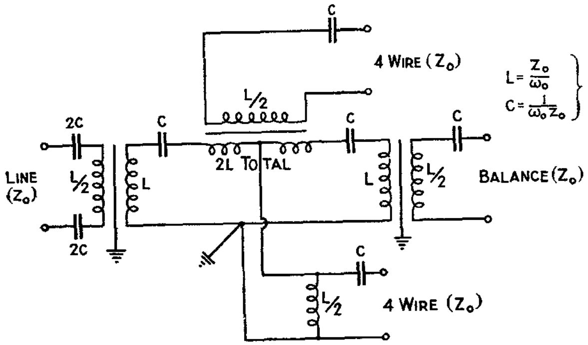
Hybrid transformers in filter circuits.

When a carrier telephone circuit is operated on a duplex basis, a carrier hybrid transformer can conveniently be built into the filter structure and its winding inductances utilised as filter elements. Once again difficulties due to resonance of high impedance transformers and to leakage inductance are avoided. The winding inductance contributes to a basic type high-pass network.

This is shown in Fig. 16 in which the connections to line, balance and 4-wire legs are all basic type filter impedances. Other sections can, of course, be added if so desired and impedances may be modified by means of the transformers.



DEVICE FOR CONNECTING AN UNBALANCED H.P FILTER TO A BALANCED LINE OR NETWORK.



USE OF A DIFFERENTIAL TRANSFORMER AS A HIGH PASS FILTER ELEMENT.

FIG. 16.

Propagation times through filter networks.

The envelope delay time of a steady state wave through any network is given by $\frac{d\alpha}{d\omega}$. In the case of some types of electric wave filters, this is easily reduced to formulæ if resistance is neglected. The effect of resistance will always be small.

Type.	Constant K.	Derived.	Remarks.
High-pass	$\frac{2000}{\omega\sqrt{x^2-1}}$	$\frac{2000\,mx}{\omega_0\sqrt{x^2-1}\left\{x^2-(1-m^2)\right\}}$	$x=\frac{\omega}{\omega_0}$
Low-pass	$\frac{2000}{\omega_0\sqrt{1-x^2}}$	$\frac{2000\,m}{\omega_0\sqrt{1-x^2}\left\{1-(1-m^2)x^2\right\}}$	$x=\frac{\omega}{\omega_0}$
Band-pass	$\frac{2000\,(1+x^2)}{\omega_m x\sqrt{n^2x^2-(x-1)^2}}$	$\frac{2000\,mn^2x(x^2+1)}{\omega_m\left\{(1-m^2)(1-x^2)^2-n^2x^2\right\}\left\{(1-2m^2)(1-x^2)^2-n^2x^2\right\}^{\frac{1}{2}}}$	$=x\quad\frac{\omega}{\omega_m}$

The above formulæ give the propagation times in milliseconds for one section. For complicated band filters it is sometimes convenient to determine $\frac{d\alpha}{d\omega}$ graphically from the phase shift curves.

As an example, Fig. 17 shows the calculated curve for a low-pass filter whose characteristics are given. At the cut-off frequency, the propagation time is infinite, if the filter is terminated with its image impedance and no resistance is present in the arms.

PART II.

The quality of reactance arms.

Every impedance element involves a combination of resistance, inductance and capacity in varying degrees, and the best is so to emphasise one of these properties that, as a first approximation, the other two may be ignored. The imperfections of the reactance arms must now be dealt with and enquiry made as to how nearly the ideal assumptions may be realised.

Condensers.

It may be assumed that, for practical purposes, choice of condensers is limited to fixed values with paper or mica dielectric. In a few cases it may be desirable to add variable condensers for trimming, but these need not be considered. The precision with which a filter is to be constructed usually determines the type of condenser.

Paper condensers of about 0.05 μF capacity can be built to a tolerance of about $\pm\ 2\frac{1}{2}\%$, but below this capacity the tolerance must increase. The power factor of good paper condensers should be about 0.003 at speech and carrier frequencies but, unless care is taken in the manufacture, it may rise to 0.01. At radio frequencies, the dissipation is rather greater. The instability of paper condensers with temperature and age, together with the wide tolerances necessary, preclude their use in high quality networks.

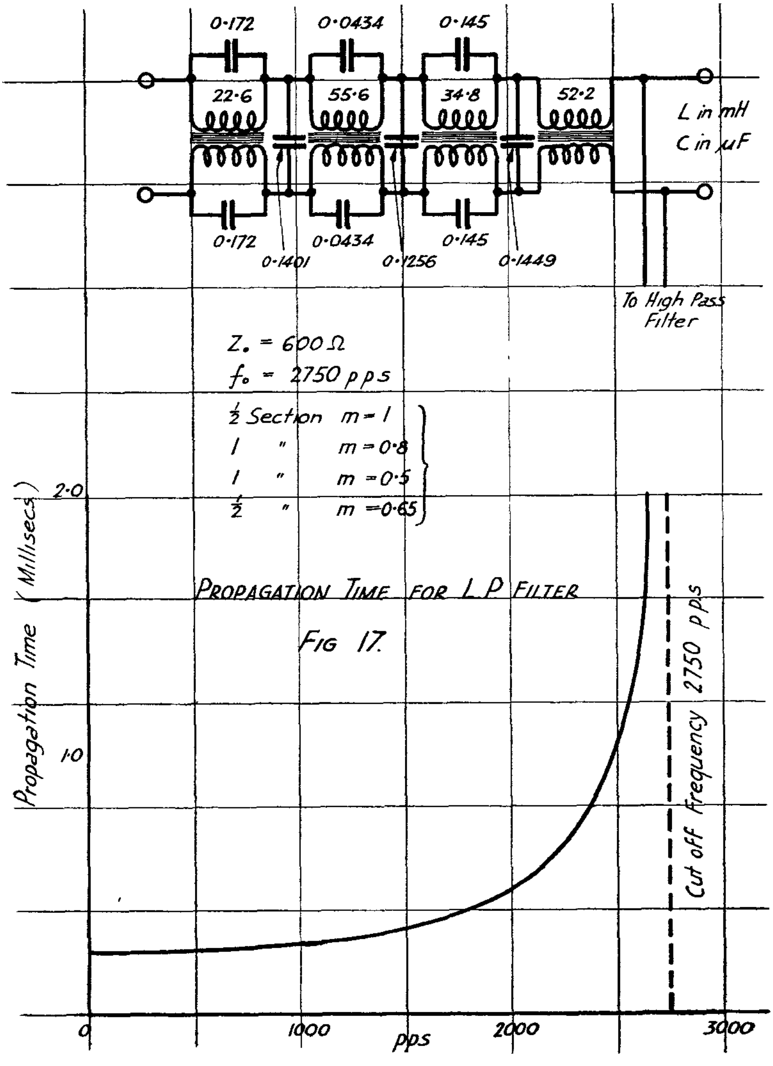


FIG. 17.

Mica condensers are usually employed where accuracy and stability are desired, and these, in their best form, consist of metallic foils (preferably copper) separated by sheets of ruby mica and clamped firmly together the whole of the overlapping area. With such condensers, high accuracy is possible, but where bulk is important, a tolerance of $\pm$ 1% or $\pm \frac{1}{2}$% is a reasonable one. High accuracy usually involves a large number of foils, as it is not considered good practice to clip the foils in adjusting. For precise adjustment of very low capacity values, it may be desirable to include a large series capacity and adjust with this.

The function of a condenser clamp is to obviate buckling of the plates (chiefly due to temperature changes), and clamps should never be used for adjustment of the capacity. To minimise the losses and to avoid all traces of intermodulation, it is sometimes desirable to use non-magnetic clamps. The use of non-metallic clamps (such as micalex) has also been suggested and this also serves to reduce the capacity between adjacent condensers where these are closely packed. For mica condensers produced on a commercial scale, a power factor of 0.0003 can be obtained at 1000 p.p.s. Even at radio frequencies, a figure of 0.001 is obtainable, although, in general, the dissipation constant will become higher with frequency.

At speech and carrier frequencies, the inductance of condensers is negligible and so there is no complication on this score. At higher frequencies the associated inductance is more commonly due to external leads than to the condensers themselves.

It may be stated that good condensers form practically ideal reactance elements, as their change of capacity, with frequency and current, is negligible.

Inductances.

The choice of inductance elements is a far more complicated matter than the choice of condensers, as their imperfections are more marked and more various. The requirements of an inductance may be summed up as follows :—

1. Its dissipation constant at the transmission frequencies must be low.

2. Its reactance should not be sensibly dependent on current.

3. In certain cases its dissipation constant should not vary with current.

4. Its external field should not be such that appreciable transference of energy to neighbouring coils,

occurs; this may depend on the mounting and proximity of the coils.

5. Its dimensions should not be inconveniently large.

The type and bulk of the coils used will depend to some extent on the operating frequency, but the coils will probably be selected from one of the following classes:—

1. Air-cored solenoids.
2. Air-cored toroids.
3. Magnetic stampings with a series air gap.
4. Dust-cored coils of the type used for loading coils.

Air-cored solenoids.

These form the cheapest class of filter inductance and, provided they are used with care, they may be very satisfactory. For a coil of reasonable size, say 20-25 cubic inches in its case, the direct-current resistance is such that the dissipation constant is high at frequencies below about 5 kc. Even at this frequency the losses are undesirably high (d = 0.02 or 0.03). Such a coil will have a considerable leakage field and must either be screened or spaced from associated coils. Screening will reduce the inductance of the coil and increase its effective resistance. Air-cored solenoids have resistance and reactance values which are independent of current, provided that non-magnetic screens are used. In order to obtain low dissipation constants at carrier frequencies, the use of stranded wire to reduce eddy current losses is desirable. In a good unscreened coil the dissipation constant may reach 0.007 at the optimum frequency, which frequency will, however, be about 30,000 p.p.s.

Air-cored toroids.

These are rather more expensive to wind than solenoids, but their external field is small. Hence, less care need be taken in screening or spacing the coils.

Magnetic stampings.

In order to obtain the necessary stability with current, it is advisable that these coils be wound on cores with a considerable series air gap. In coils intended for very low frequencies, such as sub-audio telegraphs, this may conflict with requirements of dissipation constant, but, at speech and carrier frequencies, a large gap is desirable on this account also.

The air gap should be adjusted to give the optimum dissipation constant, provided that this also gives adequate stability with current. The size of the gap will be substantially independent of the inductance required. Stampings down to 4 mils thickness are readily obtainable, but sometimes 2-mil stampings are desirable.

Loading coil cores (Dust Cores).

These consist of small particles of iron or nickel iron pressed into rings with a binding material. The permeability ranges from less than 20 to 100 according to requirements. Inductances wound on these cores have much more desirable properties than those wound on stampings and the dissipation constants are considerably less. The coils are toroidal in form and have a low external field; they may therefore be mounted in compact metal screens without much loss. For high frequencies stranded conductors are desirable and the optimum dissipation constant occurs at about 6000 p.p.s. with a permeability of 50.

Imperfections of inductance elements.

These may be summarised as follows:—

(1) Resistance.
  (*a*) D.C. resistance.
  (*b*) Eddy current resistance due to windings, core and screens.
  (*c*) Hysteresis resistance.
  (*d*) Skin effect.

(2) Self-capacity.

(3) Non-uniformity of inductance.
  (*a*) Variation with current.
  (*b*) Variation with frequency due to self-capacity.
  (*c*) Variation with frequency due to eddy currents.

In some cases these imperfections are closely related to each other. Skin effect is not important at carrier frequencies although the reduction of eddy current losses by means of insulated strands (not necessarily Litz) is frequently desirable.

Resistance of inductances.

At frequencies well below that at which self-resonance occurs, the resistance of an inductance coil is given approximately by

$$R = R_0 + (aI + b)f + cf^2$$

where R_0 = D.C. resistance
I = current through coil
f = frequency
a, b and c are constants for the coil and core.

To approximate rather more, we can neglect b and write

$$R = R_0 + aIf + cf^2$$

Of these terms, the second represents the hysteresis loss in the iron, and the third the eddy current losses in the core and winding. For a given core, it is possible to determine a hysteresis constant which may be expressed (for a given frequency) as

$$Fh^1 = \frac{\text{rise in resistance (ohms)}}{\text{change in current } (m\text{A}) \times (\text{Henries})^{3/2}}$$

This being roughly proportional to frequency, a constant for the core may be written as $Fh = \dfrac{Fh^1}{f \text{ in kilocycles}}$

In the case of dust-cored toroids this figure will be practically a true constant but, in the case of magnetic stampings, eddy currents in the iron may modify the value. In this case, some connection between the hysteresis and eddy current losses will exist.

Where eddy current losses are small, the eddy-current constant $F_e = \dfrac{\text{Resistance at zero current} - R_{dc.}}{\text{L in henries} \times f^2\,(kc)}$ will be practically independent of frequency. The only divergence will be due to the bf. term in the resistance expression. This represents the difference between static and dynamic molecular friction and its effect will be to increase the apparent value of F_e at low frequences.

Typical values of Fh and Fe are shown in Figs. 18 and 19. For the stampings, the figure shown for Fe applies only when the insulation is adequate. This is not always so if the stampings are used in the condition in which they are supplied by the makers. The lower the " constants " Fh and Fe, the more nearly are they independent of frequency. Thus in the case of a main line loading coil, Fe is 2.0 at 3000 p.p.s., and 1.7 at 30,000 p.p.s., whereas, in the case of stalloy stampings without an air gap, it is 3000 at 500 p.p.s and 800 at 5000 p.p.s. The figures given for the stampings should be treated as a guide only.

Hysteresis Constants for Various Cores.

$$Fh = \frac{\text{ohms change.}}{\text{mA change} \times L^{3/2} \times \text{kilocycles.}}$$

Core and air gap.	Cross Secn. sq. cms.	Mean Path Cms.	Fh
Mumetal, lap joints	1.61	14	1500
,, butt joint	,,	,,	100
,, 1/16 in. air gap ...	,,	,,	10
,, 1/8 ,, ,, ,, ...	,,	,,	8
,, 1/4 ,, ,, ,, ...	,,	,,	7
,, 3/8 ,, ,, ,, ...	,,	,,	6
,, 1/2 ,, ,, ,, ...	,,	,,	5
Radiometal, no air gap ...	,,	,,	900
,, 1/16 in. air gap ...	,,	,,	27
,, 1/8 ,, ,, ,, ...	,,	,,	18
,, 1/4 ,, ,, ,, ...	,,	,,	12
,, 3/8 ,, ,, ,, ...	,,	,,	9
,, 1/2 ,, ,, ,, ...	,,	,,	7
Rhometal, no air gap	,,	,,	600
,, 1/16 in. air gap ...	,,	,,	40
,, 1/8 ,, ,, ,, ...	,,	,,	32
,, 1/4 ,, ,, ,, ...	,,	,,	18
,, 3/8 ,, ,, ,, ...	,,	,,	13
,, 1/2 ,, ,, ,, ...	,,	,,	9
Stalloy, no air gap	,,	,,	2000
,, 1/16 in. air gap ...	,,	,,	150
,, 1/8 ,, ,, ,, ...	,,	,,	120
,, 1/4 ,, ,, ,, ...	,,	,,	90
,, 3/8 ,, ,, ,, ...	,,	,,	50
,, 1/2 ,, ,, ,, ...	,,	,,	35
Minor circuit loading coil ...	1.3	12.5	25
Main line loading coil	2.1	18.2	10
DU dust core	4.2	20.1	1

FIG. 18.

Direct current resistance.

Where low frequencies and/or low permeabilities are employed, direct current resistance is of importance and needs to be kept low. The approximate values of R/L obtainable with the various cores discussed are set out in Fig. 20.

Self-capacity of inductances.

The self-capacity of inductance windings varies considerably, but is chiefly dependent on the geometry of the coil. It will not change appreciably with turns, provided the core is wound full. For the coils as used on the magnetic

EDDY CURRENTS CONSTANTS FOR VARIOUS CORES.

$$Fe = \frac{\text{Added resce. at zero current}}{\text{L (henries)} \times \text{(kilocycles)}^2}$$

Core Material.	Specific Resce. Ω/cm cube.	Cross Secn. Sq. cms.	Mean Path Cms.	Fe No gap.	Fe 1/16″ gap.	Fe 1/8″ gap.	Fe 1/4″ gap.
Mumetal 14 mils ...	45	1.61	14	2100	100	95	90
„ 5 mils				400	70	65	60
Radiometal 14 mils ...	50	„	„	1300	260	230	200
„ 5 mils				300	180	160	140
Rhometal 14 mils ...	95	„	„	450	85	80	75
„ 5 mils				150	60	55	50
Stalloy 14 mils ...	35	„	„	500-2000	120	105	90
„ 5 mils				200-600	80	70	60
Main line loading coil		2.1	18.2	1.7	—	—	—
Minor circuit loading coil		1.3	12.5	2.5	—	—	—
DU dust core ...		4.2	20.1	1.2	—	—	—

NOTE.—In the case of Magnetic Stampings, these figures will vary considerably with frequency, and should be regarded as a rough indication only.

FIG. 19.

$\frac{R_{dc}}{L}$ *for various Cores* detailed in figures 17 and 18.

Core Material.	R/L, no air gap.	R/L, 1/16″ gap.	R/L, 1/8″ gap.	R/L, 1/4″ gap.
Mumetal	$1\frac{1}{2}$—4*	25	35	55
Radiometal	3—7*	30	40	60
Rhometal	4—10*	35	48	70
Stalloy	3—15*	50	70	100
Main line loading coil ($\mu = 50$)	36	—	—	—
Minor circuit loading coil ($\mu = 80$) ...	40	—	—	—
DU dust core $\mu = 28$)	60—100†	—	—	—

* Variations with Frequency and Flux Density.

† The larger figure applies to stranded wire.

FIG. 20.

stampings already discussed, the following figures are relevant:—

Single-section bobbin	... ...	120 $\mu\mu$F
Two-section bobbin	... ...	30 $\mu\mu$F
Three-section bobbin	... ...	14 $\mu\mu$F
Four-section bobbin	... ...	8 $\mu\mu$F

These figures refer to a thoroughly dry coil. The presence of a little moisture (as in a coil which is not impregnated) may increase them considerably.

In the case of toroidal dust-cored coils, the self-capacity again depends on the method of winding. In an overwound coil, designed to give a minimum of leakage inductance, the capacity is about 800 $\mu\mu$F, but if a two-section winding is used, this is reduced to about 250 $\mu\mu$F. The addition of another pair of diagonal flanges, thus forming a four-section winding, further reduces this capacity to about 80 $\mu\mu$F. A further reduction can be effected by filling the winding space with two or three layers only, but this device may only be employed at high frequencies where the direct-current resistance is unimportant.

Dust-cored versus air-cored inductances.

Examination of the foregoing data will make it clear that for high quality filters it is advisable to employ either

DISSIPATION CONSTANTS FOR VARIOUS COILS $(d = \frac{R}{\omega L})$

For low flux densities, about $I\sqrt{L} = 0.1$ (mA × henries).

Core Material.	At 1000 p.p.s.				At 10000 p.p.s.			
	No gap.	1/16″ gap.	¼″ gap.	1/4″ gap.	No gap.	1/16″ gap.	1/8″ gap.	⅛″ gap.
Mumetal 14 mils ...	0.16	0.022	0.025	0.027	1.5	0.15	0.13	0.1
,, 5 mils ...	0.06	0.015	0.02	0.025	0.5	0.06	0.05	0.04
Radiometal 14 mils ...	0.2	0.045	0.042	0.04	1.5	0.3	0.25	0.2
,, 5 mils	0.08	0.034	0.032	0.032	0.5	0.15	0.12	0.1
Rhometal 14 mils ...	0.09	0.019	0.021	0.025	0.6	0.10	0.08	0.07
,, 5 mils ...	0.03	0.015	0.017	0.020	0.2	0.05	0.045	0.04
Stalloy 14 mils ...	0.4	0.035	0.035	0.038	0.8	0.12	0.1	0.08
,, 5 mils	0.15	0.024	0.026	0.028	0.25	0.05	0.035	0.03
Main line loading coil	0.006	—	—	—	0.0035	—	—	—
Minor circuit loading coil	0.007	—	—	—	0.005	—	—	—
DU dust core ...	0.01	—	—	—	0.003	—	—	—

FIG. 21.

air-cored or dust-cored toroids. The advantages and disadvantages of the two types are set out below :—

Advantages.	Disadvantages.
Air-cored coils.	
1. Complete absence of intermodulation effects.	1. Large dissipation constants at low frequency.
2. Cheapness.	2. Bulk.
	3. Strong external field (solenoid) and necessity for spacing the coils.
Dust-cored toroids.	
1. Small size.	1. Variation of inductance and resistance with current to varying degrees.
2. Complete absence of external field when screened.	2. Cost.
3. Better dissipation constants, particularly at frequencies below 10,000 p.p.s.	
4. Convenience of mounting.	

Post Office practice.

Post Office practice has been to use dust-cored coils almost exclusively for high quality filters. In most cases minor circuit loading-coil cores have proved satisfactory, but in particularly difficult cases, where intermodulation is liable to occur, DU core material has been employed. Intermodulation between two frequencies will occur if they both pass through some network having inductance or resistance which varies with current. This may be serious where several frequency bands pass through a common filter or filter coil, as the intermodulation may give rise to cross-talk. Specific cases of the use of DU cores are carrier line filters in multichannel carrier telephone circuits and the end coils of the associated speech channels.

The stability of dust-cored coils, freedom from interference, low losses due to screens, etc., together with the marked economy of mounting space, has been such that the slightly increased initial cost has been worth while, and no reason is seen for departure from this satisfactory practice.

In those cases where air-cored coils are used, the introduction of screens and the proximity of other metallic parts cause variations in the inductance of the coils. To overcome this, new manufacturing methods have been employed in the United States, in which each inductance is made continuously variable over a range of about 3% and is adjusted inside all screens. High accuracy is claimed for this method, but the Post Office has had no experience of its use.

The tolerance allowed on filter coils made for the Post Office is usually ± 1% at a specified current. Allowing for a further ± ½% for diversity of operating conditions, this gives a possible inaccuracy of 1½%. With condensers specified to ½% tolerance, the maximum possible error in critical frequencies is ± 1%. This is not subject to any increase due to the presence of screens, etc., and may be taken as satisfactory for nearly every purpose. At 30,000 p.p.s., the highest frequency usually considered, this gives a maximum error of 300 p.p.s. in the resonance frequencies. This may be halved by suitable grouping of the components, but the production costs are increased.

Wiring of filters.

The precautions which are necessary in assembling and wiring the components will depend upon

1. The frequencies at which the filter is desired to attenuate.
2. The attenuation required.
3. The nature of the inductances used.

Where screened dust-cored toroids are used, they may be mounted as close together as may be desired, though, in practice, coils at opposite ends of a filter are separated as far as possible. For filters which attenuate by 70 to 80 decibels, or more, it is undesirable to use a cable form, though sometimes an exception is made in the case of a low cut-off, high-pass filter or in the case of filters required for such a purpose as sub-audio telegraphs. Apart from these cases there are, broadly speaking, two methods of assembly :

(1) The coils are mounted in groups, which may be arranged with a view to economy of space, and the condensers likewise. Connections are made with stiff insulated wire (such as Glazite) *via* a connection strip, the inductances being taken to one side of the tags and condensers to the other side. In some cases it has been convenient to mount all inductances on one side of a panel and all condensers on the other side. This method of wiring has been used extensively in the Post Office Research Section, and is in every way satisfactory.

(2) The coils and condensers are arranged in more or less natural sequence (the whole resembling a schematic diagram) and the leads are kept as short as possible. This is a very satisfactory arrangement, but tends to be uneconomical of panel space. Where unscreened coils are used, this method of mounting is highly desirable as the coils must be adequately spaced and, moreover, should be mounted with axes mutually at right angles as far as possible.

When unscreened coils are employed, it is usually necessary to take great care in screening the filter as a unit, in order to avoid cross-talk to other similar units. This is usually done with a copper cover, soldered on, this being desirable as the proximity of the cover modifies the inductance of the coils. The filters would not therefore operate correctly with the covers loose or removed. When screened coils are used, slip on covers may be employed, and no great care need be taken in their use. When screens are fitted these should be of low resistance (usually copper). The thickness will depend on the lowest frequency to be screened, as the optimum thickness decreases as the frequency increases. For frequencies of the order of 5000 p.p.s., 16 gauge copper is suitable and any greater thickness means wasted copper. The screen acts as a loosely coupled single turn and absorbs the energy of any stray magnetic field. Regarded from the standpoint of a power load due to the eddy currents, the losses will be zero for infinite or zero resistance.

Testing of filters.

The only tests which it is necessary to apply to a completed filter are:

(1) Insertion loss when working between the design impedances.
(2) Impedance when closed at the far end by the design impedance.
(3) Cross-talk to other units.

Except where it is necessary to pair filters for duplex circuits or where the input impedance is of particular interest, it will usually suffice to measure the insertion loss, as this will include the losses due to terminal mismatch. Assuming that the filter has the same design impedances at the two ends (Z_A and Z_B), the testing circuit will be as in Fig. 22.

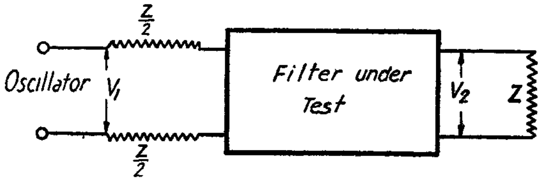


FIG. 22.

This circuit simulates exactly the losses involved in inserting the filter between a generator, having an internal impedance Z and generating a voltage V_1, and a closing impedance Z.

The total insertion loss is given by

$$20 \log_{10} \frac{V_1}{2V_2} \text{ decibels}$$

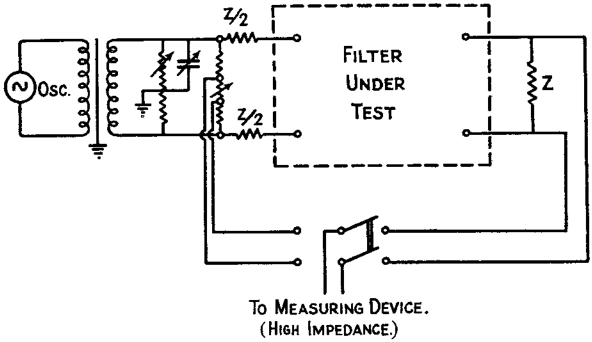
When testing an unbalanced filter, the sending end resistance should be in one leg only, the other leg (filter common) being earthed. For filters involving an impedance transformation, the correction will be obvious.

It is usual to divide the insertion loss tests into two parts, firstly, of the losses in the transmission band, in which comparison of input and output voltages may be made by means of a voltmeter, and secondly, of the losses in the attenuating bands, in which an amplifier-rectifier unit is necessary. In either case, it is usual to include a potentiometer across the input circuit and to equate a fraction of the input voltage to the output voltage. An absolute voltage calibration is not then necessary. By using a sensitive amplifier-rectifier in conjunction with a reflecting galvanometer, it is comparatively easy to measure attenuations up to 120 decibels provided that certain precautions are taken :

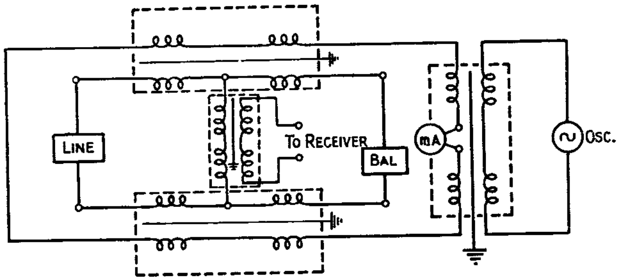
1. The oscillator and amplifier should be separated and run from separate batteries.
2. The amplifier should be effectively screened.
3. The input leads to the amplifier and all leads associated with the filter output circuit should be screened.
4. The oscillator output should be carefully balanced to earth, if necessary by means of added capacity and resistance.
5. When a high-pass filter is being measured, the harmonics should be effectively suppressed.

Provided reasonable care has been taken in arranging the components and wiring, it will be found that the measured insertion loss will agree closely with the calculated loss and the design figures will be closely realised.

When an impedance run is taken over the transmission band, it is desirable, in the case of a balanced filter, to use a balanced type of bridge. Such a bridge is shown in Fig. 23



MEASUREMENT OF FILTER INSERTION LOSS.



Transformer Bridge for Balanced Circuit Measurements.

TESTING OF FILTERS.

Fig. 23.

and consists of four balanced and screened transformers arranged to form a bridge network. This bridge has given very satisfactory service. The current through the network under test can be measured by means of an A.C. voltmeter across a resistance as indicated. This may be calibrated against a thermo-couple which takes the place of the network in a preliminary test.

APPENDIX A.

Intermodulation products.

In order to see how intermodulation arises and the magnitude of its products, it is convenient to study a single constant K, low-pass section carrying sinusoidal currents $\iota_1 \cos \omega t$ and $\iota_2 \cos \omega_2 t$ at its input terminals and terminated with its image impedance.

The current through the coil at frequency ω_1 will be $\sqrt{1 - \left(\frac{\omega_1}{\omega_0}\right)^2}\ \iota_1$ and the instantaneous resistance to frequency ω_1 due to hysteresis losses is

$$\sqrt{2}\, Fh_1 \frac{\iota_1}{\sqrt{1 - \left(\frac{\omega_1}{\omega_0}\right)^2}} \cos \omega_1 t +$$

$$\sqrt{2}\, Fh_2 \frac{\iota_2}{\sqrt{1 - \left(\frac{\omega_1}{\omega_0}\right)^2}} \cos \omega_2 t$$

where Fh_1 and Fh_2 are the hysteresis factors for the coil at frequencies ω_1 and ω_2, expressed as ohms/milliamp/coil.

The output currents due to frequency ω_1 will be

$$\frac{\sqrt{2}\, Fh_1 \iota_1^2}{4Z_0} \cos 2\omega_1 t + \frac{\sqrt{2}\, Fh_2 \iota_1 \iota_2}{2Z_0} \frac{\sqrt{1 - \left(\frac{\omega_1}{\omega_0}\right)^2}}{\sqrt{1 - \left(\frac{\omega_2}{\omega_0}\right)^2}} \cos \omega_1 t \cos \omega_2 t$$

and due to frequency ω_2,

$$\frac{\sqrt{2}\, Fh_2 \iota_2^2}{4Z_0} \cos 2\omega_2 t + \frac{\sqrt{2}\, Fh_1 \iota_1 \iota_2}{2Z_0} \frac{\sqrt{1 - \left(\frac{\omega_2}{\omega_0}\right)^2}}{\sqrt{1 - \left(\frac{\omega_1}{\omega_0}\right)^2}} \cos \omega_1 t \cos \omega_2 t$$

Magnitude of $2\omega_1 = \dfrac{\sqrt{2}\, Fh_1 \iota_1^2}{4Z_0}$

,, ,, $2\omega_2 = \dfrac{\sqrt{2}\, Fh_2 \iota_2^2}{4Z_0}$

$$,, \quad ,, \quad \begin{matrix} \omega_1 + \omega_2 \\ \text{and} \\ \omega_1 - \omega_2 \end{matrix} = \left[\frac{\sqrt{2}\,Fh_2}{4Z_0} \frac{\sqrt{1 - \left(\frac{\omega_1}{\omega_0}\right)^2}}{1 - \left(\frac{\omega_2}{\omega_0}\right)^2} + \frac{\sqrt{2}\,Fh_1}{4Z_0} \frac{\sqrt{1 - \left(\frac{\omega_2}{\omega_0}\right)^2}}{\sqrt{1 - \left(\frac{\omega_2}{\omega_0}\right)}} \right] \iota_1 \iota_2$$

Cross-talk may be set up for one or more of the following reasons:—

1. The frequency $2\omega_1$ may lie in the band of which ω_2 is a typical frequency.
2. The frequency $2\omega_2$ may be in the band of which ω_1 is a typical frequency.
3. The intermodulation product gives frequencies $(\omega_1 + \omega_2)$ and $(\omega_1 - \omega_2)$ either or both of which may lie in one of the transmission bands of which ω_1 and ω_2 are typical frequencies.

The frequency ω_1 will be modulated by ω_2 to a depth of

$$\frac{\sqrt{2}\,\iota_2}{4Z_0} \left\{ Fh_2 \frac{\sqrt{1 - \left(\frac{\omega_1}{\omega_0}\right)^2}}{\sqrt{1 - \left(\frac{\omega_2}{\omega_0}\right)^2}} + Fh_1 \frac{\sqrt{1 - \left(\frac{\omega_2}{\omega_0}\right)^2}}{\sqrt{1 - \left(\frac{\omega_0}{\omega_2}\right)}} \right\}$$

$$= \frac{\sqrt{2}\,\iota_2}{4Z_0} \left\{ Fh_2 \sqrt{\frac{0.64}{0.36}} + Fh_1 \sqrt{\frac{0.36}{0.64}} \right\}$$

assuming $\frac{\omega_1}{\omega_0} = 0.6$, $\frac{\omega_2}{\omega_0} = 0.8$

$$= \frac{\sqrt{2}\,\iota_2}{4Z_0} (1.33\,Fh_2 + 0.75\,Fh_1) = \frac{\sqrt{2}\,\iota_2}{4Z_0}\, 2.53\,Fh_1$$

If $\iota_2 = 5\ mA$, $Z_0 = 1000\Omega$, $Fh_1 = 0.3\Omega/mA$

Depth of modulation $= 1.35 \times 10^{-3}$

This is equivalent to an interference ratio of 57 decibels.

APPENDIX B.

Dissipation losses in true band-pass filters and in a high and low-pass pair arranged to form a band filter.

It has been shown by G. J. S. Little* that the losses in the transmitting band of a constant K filter are given by

$$\beta k = \frac{Z_{1k}(R_1 + R_2)}{Z_{01k}} \text{ nepers approx.}$$

where Z_{1k} is the series arm, Z_{01k} is the mid-series image impedance and R_1 and R_2 are the dissipation constants for the series and shunt arms respectively. For a derived section this has to be multiplied by a factor

$$F = \frac{m}{1 - (1 - m^2) Z_{1k}^2}$$

For high, low and band-pass sections these reduce to

Type.	β_k	F	Remarks.
High-pass	$\frac{r_1 + r_2}{\sqrt{x^2 - 1}}$	$\frac{mx^2}{x^2 - (1 - m^2)}$	$x = \frac{\omega}{\omega_0}$
Low-pass	$\frac{x(r_1 + r_2)}{\sqrt{1 - x^2}}$	$\frac{m}{1 - (1 - m^2)x^2}$	$x = \frac{\omega}{\omega_0}$
Band-pass	$\frac{2(r_1 x^2 + r_2)}{\sqrt{n^2x^2 - (1 - x^2)^2}}$	$\frac{m}{1 - (1 - m^2)\left(\frac{nx}{1 - x^2}\right)^2}$	$x = \frac{\omega}{\omega_m}$

In the above table, r is the dissipation constant of all inductances used and r_2 is a similar figure for all condensers.

Fig. 24 shows the dissipation losses in a band-pass section of the following characteristics. Cut-off frequencies 10,700 and 13,450 p.p.s. giving $n = 0.228$

m is taken as 0.6 and dissipation constants are allowed $r_1 = 0.01$, $r_2 = 0$.

For comparison purposes, a high and low-pass combination is calculated.

(*a*) In which $m = 0.6$ for each of the high-pass and low-pass sections.

(*b*) In which $m = 0.235$ for each of the high-pass and low-pass sections, thus giving the resonance peaks

* I.P.O.E.E. Paper No. 143, " Electric Wave Filters," by G. J. S. Little.

in approximately the same position as for the band-pass filter.

The advantage which is exhibited by the high and low-pass filter, in this respect is considerably more marked as the ratio of band width to mean frequency (n) is increased.

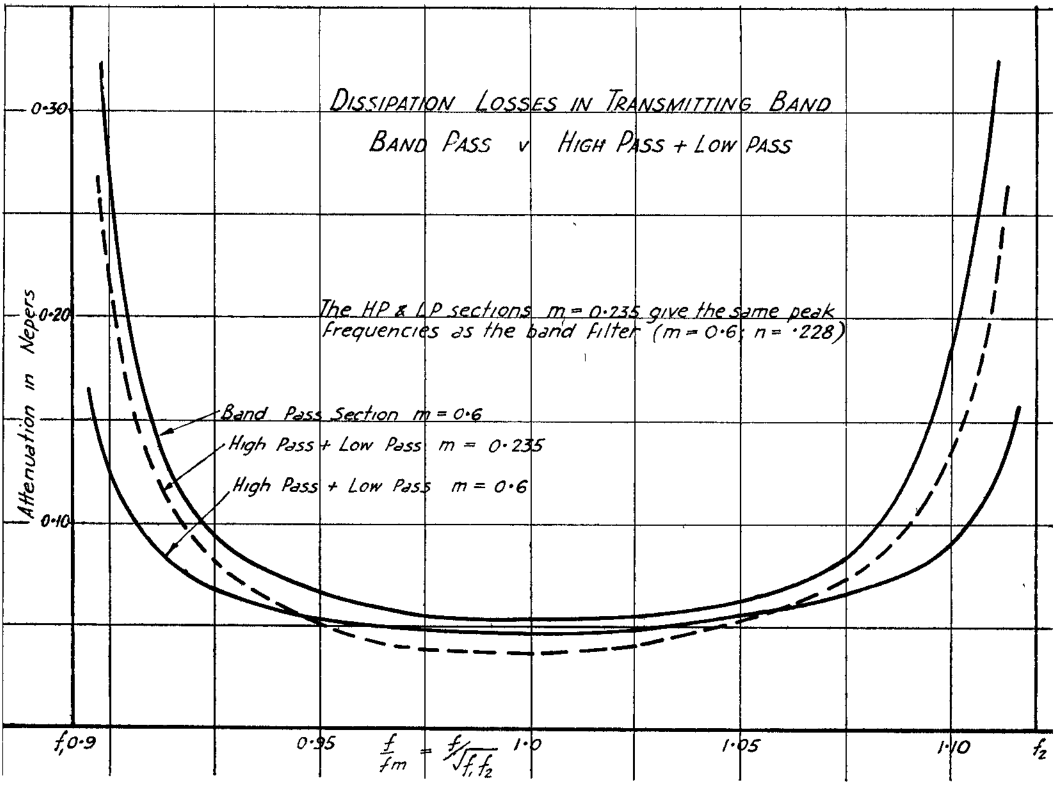
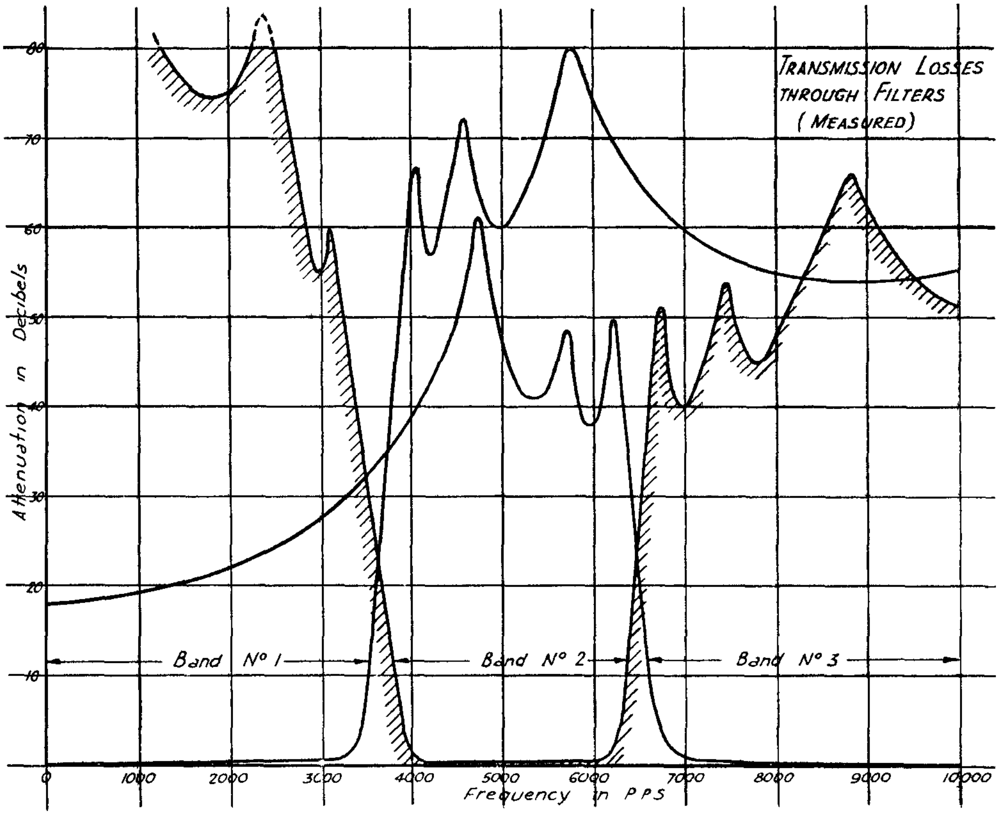


FIG. 24.



ATTENUATIONS OF A TYPICAL SET OF CARRIER FILTERS.

FIG. 25.

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DISCUSSION.

Capt. A. C. Timmis :—

The subject of the paper is necessarily difficult, but is becoming more and more important both in telephony and telegraphy. It might be useful, therefore, if I amplified the general statement of the author regarding the uses of filters.

The earliest use of filters outside the laboratory was in two-wire repeaters. Filters were inserted to cut down the gain at the higher audio-frequencies where a loaded line is difficult to balance. These filters, at first, were of the mutual inductance type, as shown in Fig. 14. With the advent of carrier telephony and voice-frequency telegraphy a wider field was opened up for the application of filters, and lately of course band-pass filters have been used in radio receivers. The chief application of filters is to carrier telephony, where they constitute perhaps the most important part of the apparatus. Carrier systems are now being introduced rapidly in this country, on aerial lines and submarine cables, but an even wider application is, I think, made possible by the use of four-wire carrier working on the main underground cables. At present, of course, most of our underground cable circuits are four-wire, but by using very light loading it is possible to superimpose on each circuit a four-wire carrier, thus obtaining two circuits in place of one. The relative costs have been worked out carefully and show a saving of 50% in favour of the carrier method, *i.e.*, three circuits can be obtained for the cost of two.

The indications are that the main cables of the future will comprise circuits of that type, the system having proved quite satisfactory in preliminary tests.

I have departed somewhat from the strict limits of the paper, but in dealing with such a highly specialised subject as filters it is desirable that the practical applications should be emphasised.

There is one specific point I would like to refer to :— When an inductance coil is enclosed in a copper screen it is found that the lower the resistance of the screen, the lower is the effective resistance of the coil at high frequencies. The screen may be regarded as the loosely coupled secondary of a transformer whose primary is the inductance coil, the secondary being closed by a resistance (R) which depends on the resistance of the screen. If R is very high the energy loss $\left(C^2R \text{ or } \frac{E^2}{R} \right)$ will be small because the current will be small; if R is zero there will be no volts across it and therefore no energy loss. Thus, in the extreme cases the energy loss (which appears as effective resistance) is zero. If we plot effective resistance against R the curve must rise to a maximum at some intermediate value of R. Under the conditions met with so far, it appears that the value of R lies somewhere between zero and this maximum. The practical conclusion is that fairly thick copper should be used for screening coils intended for high frequencies.

A similar conclusion appears to be justified in the analagous case of screened cable conductors; for example, it has been found that copper tape produces much less effective resistance than brass tape of the same thickness when used to screen submarine cable conductors. Also, increasing the thickness of the copper tape still further reduces the effective resistance.

Mr. A. W. Montgomery :—

In his admirable paper Mr. Halsey expressed a preference for the use of copper foil in condensers. We have found by experiment that a more suitable and efficient condenser results from the use of " tin " foil which, of course, normally contains a proportion of lead. A more solid condenser unit is obtained, and certainly for frequencies up to, say, 150 kC., a condenser which remains constant with time and has very low losses is in consequence produced.

It is appreciated that in a paper which includes such a wide range of practical detail it is impossible to mention everything, yet I think it is worth directing attention to the use of impedance transforming networks and filters. It frequently happens that the practical realisation, under manufacturing conditions, of coils and condensers of the inductance and capacity values found by calculation is extremely difficult; that is to say capacity and inductance values may be too high or too low for convenient manufacture of apparatus. In such cases, by the judicious use of an impedance transformation inside the filter, the capacity and inductance values may be changed to others which lead to more satisfactory manufacturing conditions. An instance of this may be found in the filters used in the voice frequency telegraph system described in the *Post Office Electrical Engineers' Journal* for April, 1932.

Practical tricks of this type are of more value for manufacture than for their theoretical importance; of course, while it is fairly easy, given the time, to design and construct a single filter, it is quite a different matter to make a number of filters to the same design and limits under commercial manufacturing conditions.

Mr. H. Stanesby :—

I am afraid I disagree with Mr. Halsey's statement that the substitution of a band-pass filter by low-pass and high-pass filters in series is best employed when the passing band is relatively wide. One of the most important cases where this substitution may well be employed, is in the design of a filter which is required to pass a particularly narrow band of frequencies, and to introduce large values of attenuation for frequencies immediately outside the passing band. If an attempt be made to employ the normal band-pass filter structures or their equivalent networks for this purpose, it will usually be found that the values of inductance and capacitance required are either so small or so large as to be completely impracticable. This was pointed out by Colonel Lee in his address to the Wireless Section of the Institution of Electrical Engineers in November, 1927.

In the Radio Section we find this substitution of considerable value, and Fig. 26 shows the transmission-frequency characteristic of a filter that has recently been designed on these lines. Another point which may be of some interest is the way in which this transmission characteristic is improved by connecting it in series with a band-pass filter the image impedances of which are matched only at the extreme edges of the passing band.

Fig. 27 shows the transmission-frequency characteristic of the band-pass filter which consists of two sections, separated by a valve. This serves to increase the attenuation where it was least for the preceding filter and at the same time to equalise the transmission within the passing band. The overall frequency characteristic of the complete system is given in Fig. 28.

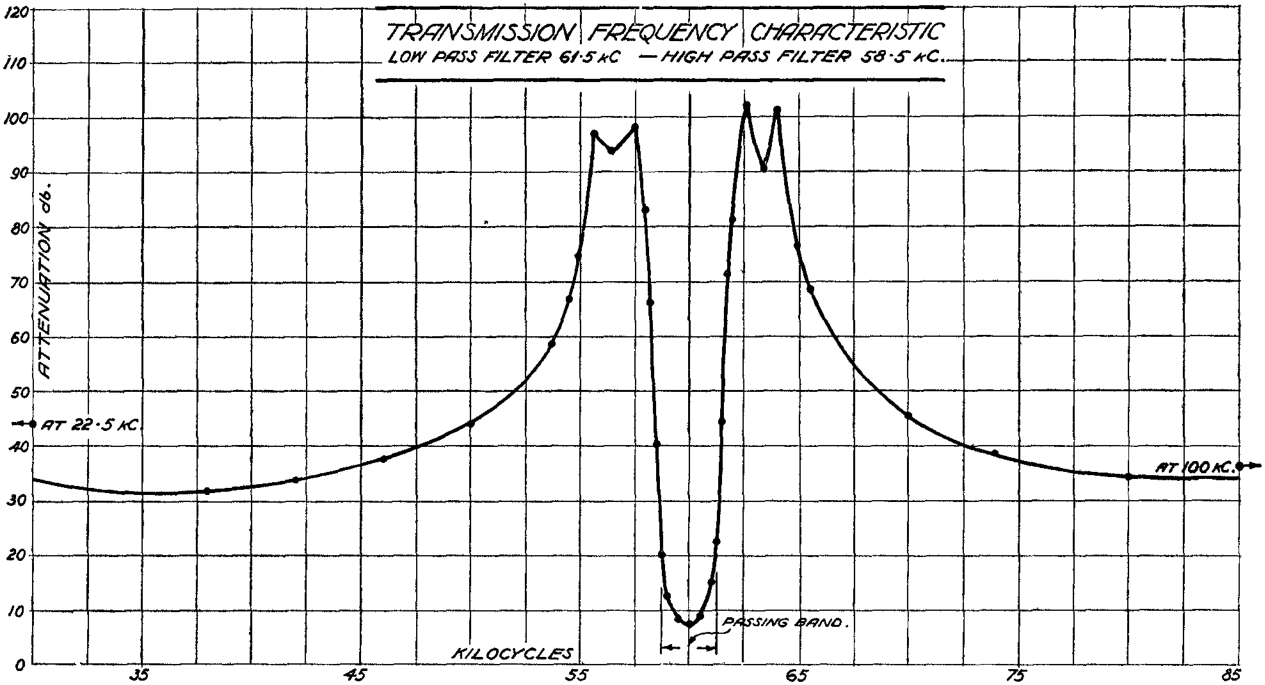

TRANSMISSION FREQUENCY CHARACTERISTIC
LOW PASS FILTER 61·5 KC — HIGH PASS FILTER 58·5 KC.
ATTENUATION db.
AT 22·5 KC.
AT 100 KC.
PASSING BAND.
KILOCYCLES
120
110
100
90
80
70
60
50
40
30
20
10
0
35
45
55
65
75
85

FIG. 26.

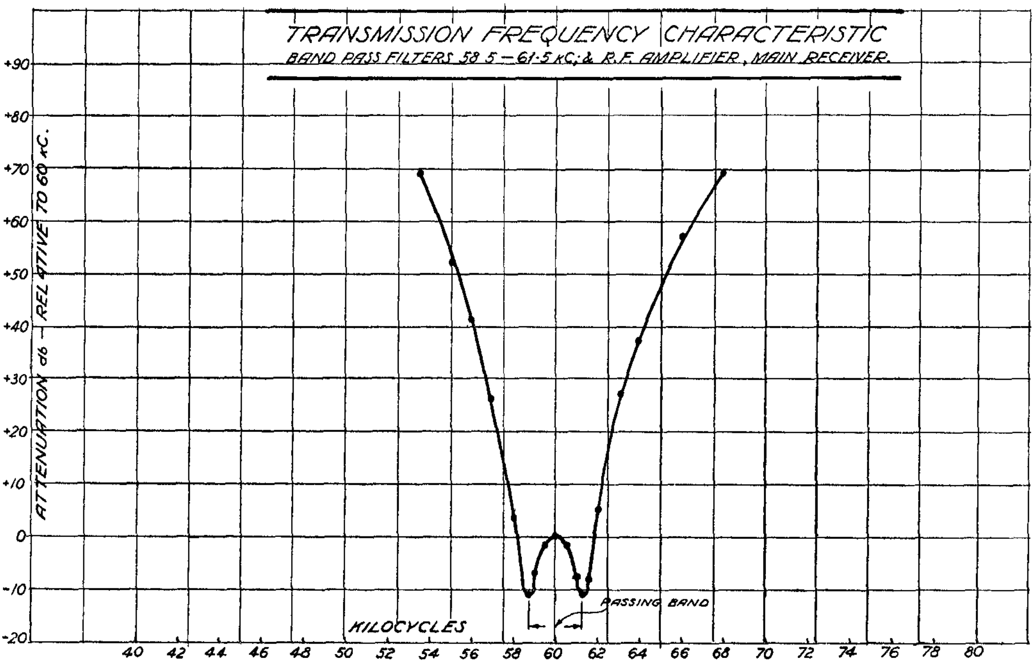
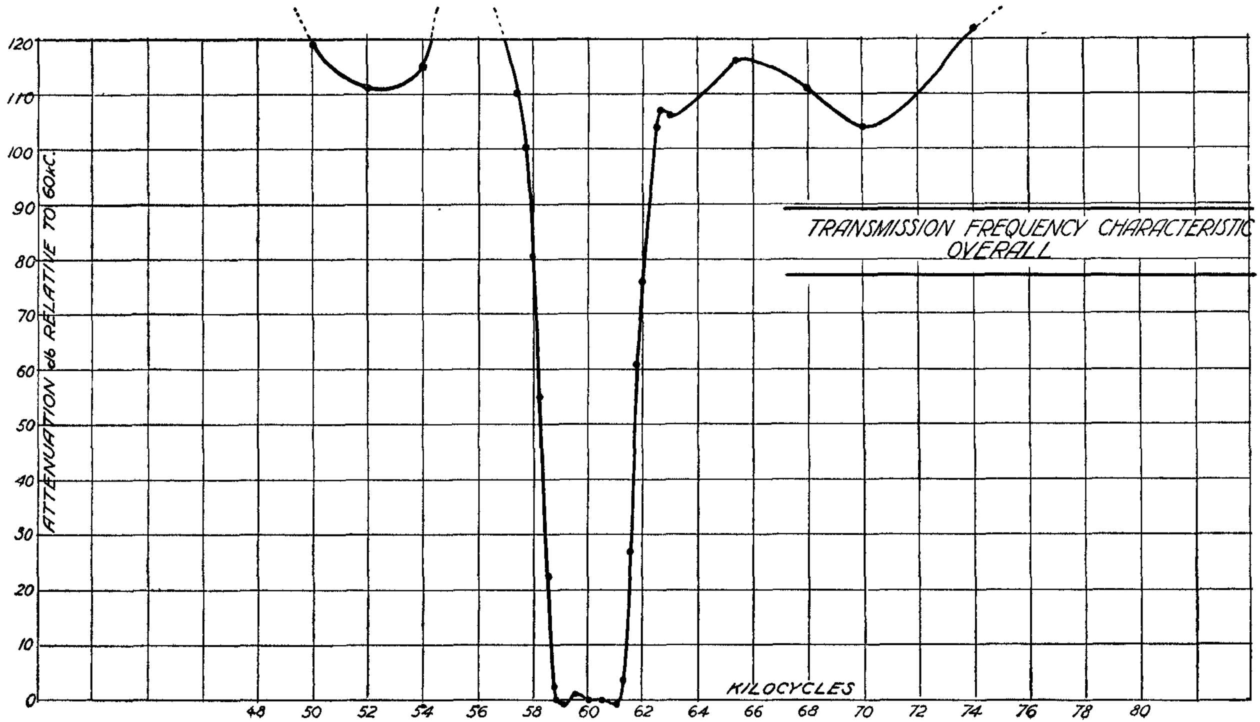


FIG. 27.

Mr. Halsey refers to the use of solenoid coils on carrier frequencies, but it appears to me that it is preferable both on account of their small size and their improved dissipation constant, to use slab, or wave-wound coils.

At the higher radio frequencies, solenoid coils have in general a lower dissipation constant than the corresponding slab coils, even though a greater length of wire is used to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ATTENUATION db RELATIVE TO 60kC.
TRANSMISSION FREQUENCY CHARACTERISTIC
OVERALL
KILOCYCLES
48
50
52
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56
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62
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68
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72
74
76
78
80

FIG. 28.

obtain the same inductance value. This is due to the fact that at these frequencies eddy current losses are so great that it is undesirable to pack a large number of turns in a small space and so to subject them to a concentrated field.

At the lower frequencies at present used for carrier work, a large amount of the coil resistance is due to the straight wire resistance of the conductor, and the increase in resistance due to winding the wire into a coil is not as great as it is at the higher frequencies. It therefore becomes more important to reduce the length of wire used, and so to approach that shape of coil which gives the largest inductance for a given length of wire, which is of course that of the slab coil.

These considerations are confirmed by results. Using slab coils it has been my experience that there is no difficulty in obtaining dissipation constants of the order of 0.005 at a frequency of 30,000 cycles per second, which compares favourably with the optimum figure of 0.007 quoted by Mr. Halsey.

Mr. D. C. Espley :—

I would like to defend the new theory of transmission network design because I think that it gives us methods of calculation which are very much more flexible and direct than those due to either Campbell or Zobel.

Having settled the requirements for a filter it is now possible to carry out the design without the trial and error methods so familiarly associated with Zobel networks. We can start off with specified impedance and propagation characteristics and the network form can be selected almost immediately from available design data. The impedance and propagation constant characteristics can be given forms approximating as closely as desired to the ideal, whilst at the same time the two characteristics may be chosen absolutely independently in the same filter.

The beauty of the method lies in the fact that in the initial stages of a design we can say that—

(*a*) The impedance over any desired fraction of the pass range shall be a resistance which does not deviate by more than any required small amount from the value of the nominal terminal impedances.

(*b*) The attenuation beyond a point, just outside the pass range, shall be always greater than any required minimum value. The proximity of this point (or points) to the cut-off frequencies can be controlled in the design.

Another advantage of this new theory is that very often there is some saving in the number of circuit elements. This may be illustrated by the fact that in some Zobel networks the requirements for image impedances demand the use of complicated half section terminations which do not contribute appreciably to the overall attenuation.

One disadvantage is that up to the present the available information on the subject is obscure and not presented in the most straightforward manner.

Mr. G. J. S. LITTLE :—

The statement of the relation between time of propagation and the slope of the phase shift curve of a line or filter, which is given in the paper at the end of Part I, is generally accepted. Referring, for simplicity, to a low-pass or high-pass filter, as the cut-off frequency is approached from within the transmitting band $\frac{d\omega}{d\alpha}$ increases rapidly and at the cut-off frequency becomes infinite. At first sight one appears to be justified in drawing the conclusion that at the cut-off frequency in a resistanceless filter, the accepted relation would give the time of propagation as infinitely great, but throughout the attenuating range of a resistanceless filter (apart from points of infinite attenuation) $\frac{d\alpha}{d\omega} = 0$.

Hence in approaching the cut-off frequency from within the attenuating range $\frac{d\alpha}{d\omega}$ at the cut-off frequency might be considered to be zero.

Of more interest than this apparent contradiction due to the right angle in the phase shift curve at the cut-off frequency is the inference, if the usually accepted theory is applied, that throughout the attenuating range of a resistanceless filter, the time of propagation is nil and in a practical case such as, for instance, a loaded line or actual filter, extremely small. This result, one feels, is unlikely to be correct and possibly the accepted relation is approximate and is valid only in cases in which attenuation is approximately uniform with respect to frequency. It may be noted that the following possible relation (in which β = attenuation per unit length),

viz., speed of propagation = $\frac{d\alpha}{d\omega} + \frac{d\beta}{d\omega}$,

since both $\frac{d\alpha}{d\omega}$ and $\frac{d\beta}{d\omega}$ are zero or infinite at the cut-off frequency in the resistanceless case, according to the direction of approach, would give a time of propagation rising to an infinite value at cut-off and then falling.

Mr. J. G. STRAW :—

Reference has been made to the use of D.U. coils where there was the possibility of cross modulation.

I should like to know whether the author can give us any further information on the apparently particularly desirable qualities of that material.

Mr. D. A. LEY :—

There are several limitations in the design of filters of the type we have just now had outlined to us, and mention should be made of the two most important of these.

Firstly, when designing a filter to be built up of a number of sections, not only must each section give a desired attenuation characteristic, but care must be taken to see that all sections possess the same characteristic impedance. This is achieved at the expense of the elements employed in the circuit. This characteristic impedance however may not necessarily conform to requirements, and special end sections have to be added whose functions are to give an approximation to the impedance required at the terminals of the network, again employing extra elements. If the filter were designed as a four-terminal network comprising one section, it could be designed to have any required attenuation characteristic and impedance characteristic employing in general fewer elements than the corresponding ladder type filter. There are thus more economical solutions of the problem under discussion than that given by the ladder type of filter.

The second limitation occurs when attempts are made to use ladder type filters at higher frequencies. It will be found that unless the band width of a filter is increased, the values of inductances in the series arms of the network do not alter. This leads to the employment of inductances of the order of 70 mHs. at, say, high carrier frequencies, and the design of such coils to have good power factors and high resonance points present difficulties to the manufacturer. It should be noted, however, that this difficulty only presents itself when it is essential to employ as part of the design the mid-series image impedance of the prototype section.

Mr. C. A. Beer :—

The late Mr. Shepherd in the year 1913 described the relation of a filter and a loaded line, and I think it is very interesting to notice, if you take the position before and after loading, the effect of adding the coils. Assuming the conductor is resistanceless then before you add the coil, the impedance is wholly reactive. When the coils are added the characteristic impedance in a properly loaded line rapidly assumes an angle as near as possible to zero, *i.e.*, it becomes non-reactive. Furthermore a line or filter has to be a channel of communication. Since we have to get a certain amount of energy into receiving the 600 ohms termination, the sending impedance cannot be reactive, as no energy would then go into the circuit, and, too, I think there is some interest attached to the fact that pure coils and condensers cannot consume any steady state energy. Therefore all the energy that goes into a filter, (useful energy), must come out. One can express it in an alternative way by saying that the transmission time was infinite.

I wondered whether Mr. Halsey, possibly on the lines suggested by Mr. Espley, thinks it possible to attempt to recast his ideas or to look at some of our normal filter theories from a different standpoint, *i.e.*, the adjustment of sending impedance modulus, and/or angle, so that the input power is always equal to the output power plus any adjustment on account of resistance components unavoidably present in the coils and condensers of filter circuits.

Mr. R. J. Halsey :—

Capt. Timmis pursued a rather interesting point, in the matter of the resistance of screens which are used on these inductances, but I think we can really sum the whole matter up in a few words by saying that the maximum absorption of energy will take place when the screen is coupled to the coil itself on a characteristic impedance basis in which case the resistance is such that it will absorb maximum power from the coil. If the working condition is on the high resistance side of that point, an increase in resistance will mean that the losses go down and if on the low resistance side, they go up, and therefore the curve shown by Capt. Timmis of losses v. resistance simply involves locating the point at which maximum power can be absorbed from a circuit with a given coupling.

Mr. Montgomery made mention of the use of tinfoil with copper with condensers, and I must say the use of tinfoil is new to me. It is quite interesting to hear that tinfoil can give better results than copper.

As regards the impedance transforming sections, I must say I quite agree with him that they can be very valuable and may save a lot in the cost of condensers, but unfortunately it has been necessary to omit many such variations in a paper of this scope.

Mr. Stanesby's remarks about high and low-pass filters being used at 60,000 cycles are certainly rather interesting. I had not considered the problem at such high frequencies, and I quite see his point that it may be quite impossible to realise suitable inductance at those frequencies. However, I think that up to about 30,000 cycles my statement was fairly true. I do not think many of us would consider the use of a high and low pass pair at frequencies up to 30 kilo cycles, at any rate, but with Mr. Stanesby's experience as a guide, we shall have to look into the matter.

The use of slab coils instead of solenoids has perhaps rather escaped our attention for the simple reason that we have really concentrated on the use of dust cored coils and seldom use solenoids of any description, and in speaking of these I had in mind slab coils as well.

I was expecting some criticism from certain directions on the general method of tackling these filters, but the Zobel method has become very common. Most people seem to have some understanding of this method, but the new methods propounded by Dr. Cauer are not yet well known. We have not had any experience of the new methods and I can therefore add little to my earlier remarks. I dare say there are certain advantages and I appreciate the point made later by Mr. Ley in connection with the waste of elements. Of course I have read of some of the works of Dr. Cauer, in which these things are mentioned, but I have not had the time to delve into the mathematics. To the best of my knowledge the only exhaustive work is in German and, as that is not one of my strong points, I am rather at a loss.

Mr. Little is right about the complex functions at the peak frequencies of the filter sections, and I agree that they are simple and not complex functions. I was referring to points not exactly at resonance, but a little way off.

The statement about the capacity of the inductances of course is not absolutely true, but one has to make certain generalities in cases like this. It is approximately true that with a given core, and assuming the coil is wound full, that the self-capacity of the coil depends only on the geometry and not the actual inductance. As a matter of fact, the capacity is principally accounted for by the direct capacity between the inner and outer layers of the coil. As an example I have figures showing that the self-capacities of coils wound in a similar manner on similar cores, range from 150 $\mu\mu F$ in the case of a 5 mH coil to 180 $\mu\mu F$ in the case of a 50 mH coil, the capacity increasing uniformly with the inductance.

In speaking of the significance of $\frac{d\alpha}{d\omega}$ I think Mr. Little is opening a rather large subject. If we pass the cut-off frequency of the filter, theoretically at any rate, we do not get any power through it. The only conditions under which we can get power transmitted through the filter is when the termination is non-reactive and, in that case, of course, the whole of the properties of the filter are slightly upset and the statements relative to infinite propagation time do not hold. I think it can be assumed that all the time the filter is definitely transmitting a frequency, its propagation time is not infinite, otherwise we need not be concerned whether it is infinite or not.

Mr. Beer's conception of the working of a filter from the impedance viewpoint is not altogether a strange one. It is often quoted in general considerations of four-terminal networks in which the losses can be considered purely from the point of view of the reflection loss between the input impedance, which itself is a rolling curve, and the transmitter impedance itself. As far as the losses in the transmission band are concerned, I do not think his conception will lead us any further than the usual conception already described.

As regards the peculiar properties of D.U. core material, I am afraid that is a little outside my scope. I do not know why D.U. material is as good as it is. Our own attempts to imitate it have been abortive. D.U., in addition to its excellent hysteresis properties, has good eddy current properties, so that it is possible to get dissipation constants of about .004 on D.U. even with considerable current.

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