No. 173

The Institution of Post Office Electrical Engineers.

Audio-Frequency Ripple from D.C. Power Supplies in Communication Engineering

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A Paper read before the London Centre on the 14th February, 1939.

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SUMMARY.

Audio-Frequency Ripple from D.C. Power Supplies in Communication Engineering.

I. Introduction.

Owing to the heavy current consumption at large modern telephone exchanges, particularly those at which V.F. signalling apparatus is to be installed, it was decided that the batteries should float across the output of the generators which were used previously for charging purposes only. This will result in a reduction of the size of the batteries required for the heavy load.

Again, with the successful development of standby power plant, which includes the automatic starting of a prime mover in the event of a mains failure and having regard to the disproportionate cost of repeater station batteries, if operated on the charge-discharge principle, and, further, the growth in the number of unattended repeater stations, it has been decided that in future the batteries at these stations should normally float across the output of generators or rectifiers.

Some smoothing equipment connected between the generator and battery is generally necessary in order to reduce the undesirable effects of audio-frequency fluctuations from the generator. In the past, a single choke was used. No actual values of inductance were specified for the choke. The specification was met if the voltage drop with D.C. across the choke did not exceed 1 volt, if the iron used in the core was not saturated by the D.C. ampere turns and if the extraneous field from the choke was sufficiently small. As a result of the investigations described in the present paper, smoothing circuits can be designed and the required performance of the components can be specified and checked by comparatively simple measurements.

It must be mentioned that other apparatus such as motors, interrupters, oscillators, etc., operating from a D.C. supply must be regarded as sources of interference, and steps should be taken to reduce any ill effects arising from them. Hence, when the maximum noise which can be permitted across the batteries has been determined, it will be necessary to specify the maximum contribution of each source, since generally the R.M.S. law of addition will apply for such indiscriminate noises.

2. Noise, its effect and measurement.

The measurement of the magnitude of a pure tone and its effect on conversation is comparatively an easy matter. When several pure tones having random relationships as regards magnitude and frequency occur simultaneously, the result is a noise and this will be continuous if the individual components are maintained. The co-relation of the results of measurements made by simple means and the actual effects of such a complex disturbance is extremely involved. However, approximations can be made. Obviously these can only be regarded as a temporary expedient and must be reviewed as science progresses.

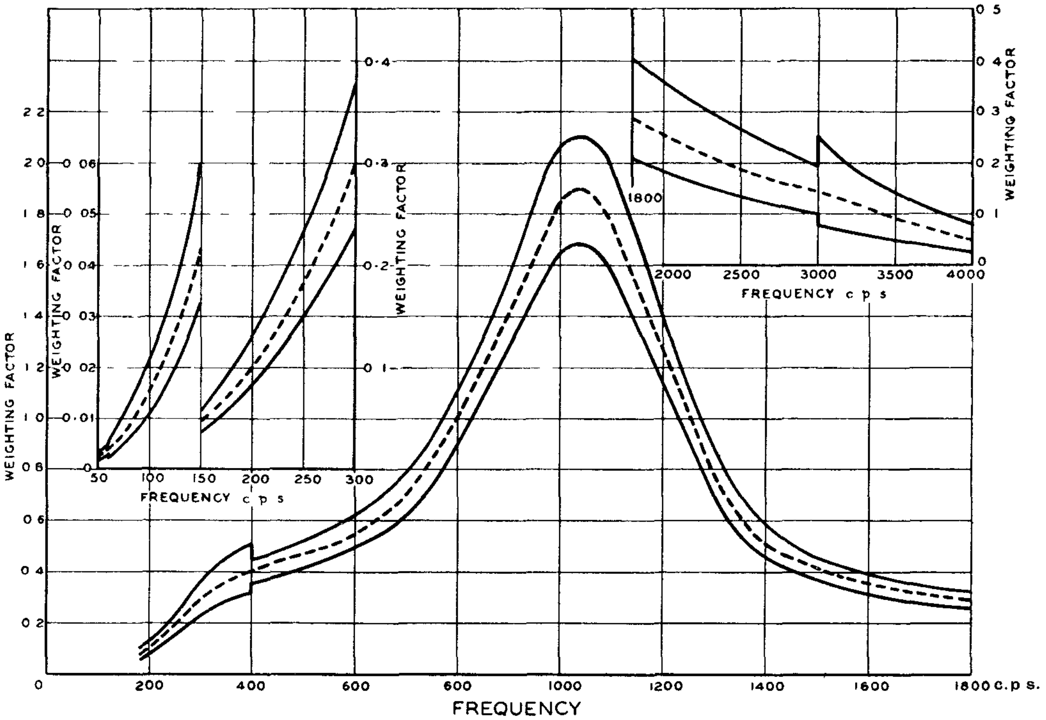

Fig. 1.—C.C.I.F. Weighting Curve and Limits.

Noise in a telephone circuit results from several causes and it is desirable that the contribution from each should be as small as possible and that a common method of measurement should apply in all cases. At the present time line noise, caused by induction from electric power supply systems is measured by means of a circuit noise-meter[1]. In this instrument the magnitude of a noise is expressed as that voltage of a pure tone at a frequency of 800 c/s which will produce the same impairment as the noise. Since the effect of a disturbance will depend, inter alia, on its frequency the C.C.I.F. has specified the response characteristic of the circuit noise-meter so that each frequency is given a definite weighting. The weighting curve is reproduced in Fig. **1**.

The noise produced in the receiver of a subscriber's instrument, due to the power supply at exchanges or repeater stations, is of the same type as that from power induction and hence the circuit noise-meter can be used for its measurement.

3. Permissible Noise Voltages across Batteries.

When batteries are floated across rectifiers or D.C. generators the maximum values (measured by psophometer) which are at present specified for noise P.D. across the batteries are as follows:—

(i) Exchanges.
    50 V supply ... ... 2 mV.
(ii) Repeater Stations.
    (*a*) 24 V. "A" supply .. 0.5 mV.
    (*b*) 130 V. "B" supply... 5 mV.
    (originally 7 mV).

It should be emphasised that these are the maximum values permitted and, hence, if the output is provided by operating several machines in parallel, the contribution from each machine should not exceed the above values divided by the square root of the number of machines used, since, in general, the R.M.S. law of addition will apply.

As regards (i), this limit was fixed as the result of actual observations made at the Sloane Exchange by a number of representatives from different branches of the E.-in-C.'s Office. The source of noise was one of the generators normally used for charging.

The ratios of the voltage across the subscriber's receiver to the voltage across the exchange battery for the circuit shown in Fig. **2** are given in the curve of Fig. **3**. It will be seen that any modification of the weighting curve of the circuit noise-meter, when used for measuring noise across an exchange battery, will only affect the lower frequencies, and these have already a small weighting factor. Further it will be observed that 2 mV at 800 c/s across the battery terminals will only cause 0.084 mV across the receiver.

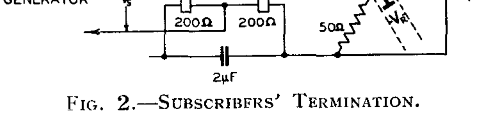

Fig. 2.—Subscribers' Termination.

The limits which must be specified under (ii) will be governed to a large extent by the decoupling arrangements incorporated in the repeaters themselves, and by the type of repeater in use. It might be argued that the instrument used for measuring the noise across repeater station batteries should have a weighting curve different from that of the circuit noise-meter as the latter is used for measurements across the line terminals. However, different methods of decoupling are used and it is undesirable that a modification should be introduced to cater for each one. For the present standard types of audio-frequency repeater it has been found that, assuming the batteries are the only source of noise, the noise output is almost entirely due, and is of the same order as the noise voltage across the "A" battery.

4. Floating Battery Power Equipment.

Apart from its D.C., E.M.F. and internal resistance, any source of D.C. supply can be regarded as a noise E.M.F. in series with an impedance. In floating battery power equipment the D.C. generator or rectifier can be considered as a source of noise, consisting of an E.M.F. in series with an internal impedance, coupled to the load by means of an electrical network. A battery forms part of this network in order that the D.C. load may be maintained under conditions of supply failure.

The noise P.D. which is produced across the load will be governed by—

(i) The magnitude and frequency of the noise E.M.F.
(ii) The internal impedance of the source.
(iii) The impedance of the battery circuit.
(This is shunted across the load impedance which is usually much greater in magnitude.)

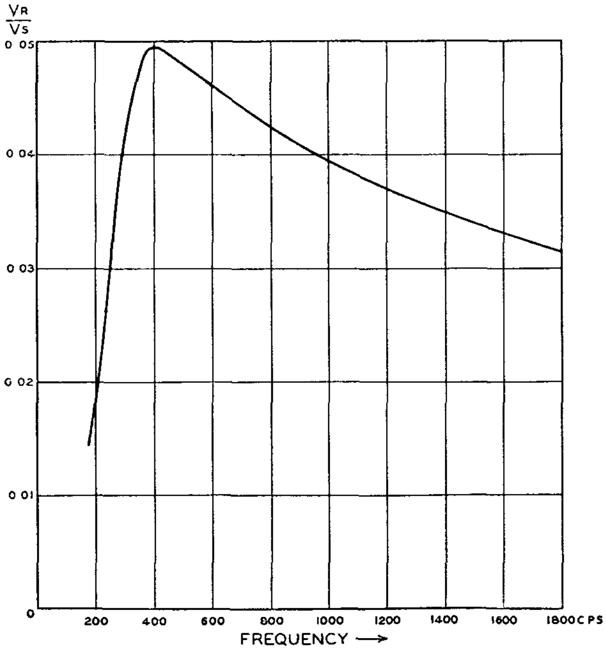

Fig. 3.—Attenuation of Subscribers' Termination.

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[1] See Bibliography.

(iv) The electrical constants of any smoothing circuit connected between the source and the battery.

5. Noise E.M.F. of D.C. Generators and Rectifiers.

(1) D.C. Generators.

The composition of the noise E.M.F. from a D.C. generator is usually extremely complex, since it contains components at the following frequencies[2]:—

(*a*) The fundamental frequency, *f*, generated in the armature and its harmonics 2*f*, 3*f*, etc.
(*b*) The slot ripple frequency and its harmonics.
(*c*) The commutator ripple frequency and its harmonics.

Analyses taken on a particular machine are given in Table 1. These are representative of measurements made on a variety of D.C. generators. It will be seen that the noise E.M.F. increases with D.C. load and that this increase is almost entirely due to slot ripple.

TABLE 1.

Analysis of Noise E.M.F. from D.C. Generator.

(Details of Generator-Shunt field ; 13/15 A, 175/131 V, 1500/1440 r.p.m. ; 4 poles ; 4 interpoles ; 32 skewed slots ; wavewound armature ; 128 commutator segments.)

No load.			Full load.			Remarks.
f	Ripple E.M.F.		*f*	Ripple E.M.F.		
	Direct.	Weighted.		Direct.	Weighted	
cycles per sec.	mV.	mV.	cycles per sec.	mV.	mV.	
50	—	—	48	—	—	Fundamental.
100	227	3.4	96	116	1.85	
150	73	3.3	144	10	0.41	
200	16.7	1.75	192	27	2.43	
250	15	3.0	240	7	1.19	
300	43	12.9	288	32	8.3	
350	16	5.6	336	28	9.5	
400	22.7	9.1	384	11.5	4.6	
450	4.2	1.83	432	—	—	
500	7.5	3.54	480	10.5	4.8	
550	5.9	3.04	528	—	—	
600	15.5	8.7	576	—	—	
650	12.3	7.9	624	—	—	
700	5.7	4.0	672	—	—	
800	125	125	768	1750	1590	Slot Ripple.
1600	57	20.1	1536	41.6	14.9	Commutator Ripple.
3200	11	1.21	3072	54	7.0	
R.M.S.		128 mV.			1590 mV.	
Circuit Noise-meter reading.		140 mV.			1840 mV.	

A series of oscillograms taken on the same machine has been reproduced in Fig. **4**. The fundamental ripple was obtained by means of a low-pass filter connected between the terminals of the machine and oscillograph. A band pass filter enabled oscillograms of the slot ripple to be taken.

From the above it will be apparent that the noise E.M.F. at full load is almost entirely due to slot ripple.

Now the slot ripple E.M.F. is generated in (i) the armature, (ii) the shunt field winding, (iii) the interpole field winding of a shunt wound D.C. machine.

With an open slot armature, the generator is effectively an alternator having salient poles on the rotor. Hence, an alternating E.M.F. will be generated in the polar windings. On a particular self-excited shunt wound machine (20 volts, 70 amps) the ripple E.M.F. at full load was 200 mV in the armature and 6,000 mV in the shunt field.

The effective ripple E.M.F. from the generator was 200 mV, which is equal to that generated in the armature alone. This is to be expected since, although the E.M.F. generated in the field winding is considerably greater than that in the armature, the shunt field winding has such a high impedance (its inductance was 45.7 *m*H) as compared with that of the armature (inductance 0.32 *m*H). The actual ripple P.D. across the machine terminals due to the E.M.F. generated in the shunt field winding was only 30 mV (calculated).

Since the number of turns on an interpole or commutating pole winding (which will be connected in series with the armature) will be few, the slot ripple E.M.F. induced in them will be small.

In order to determine the origin of slot ripple, the shunt field winding of the D.C. generator in a motor generator set was disconnected. Direct current was fed into the generator armature via a suitable choke and the machine was rotated at normal speed. The noise voltage across the armature was measured at various values of armature current. The results

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[2] See Bibliography.

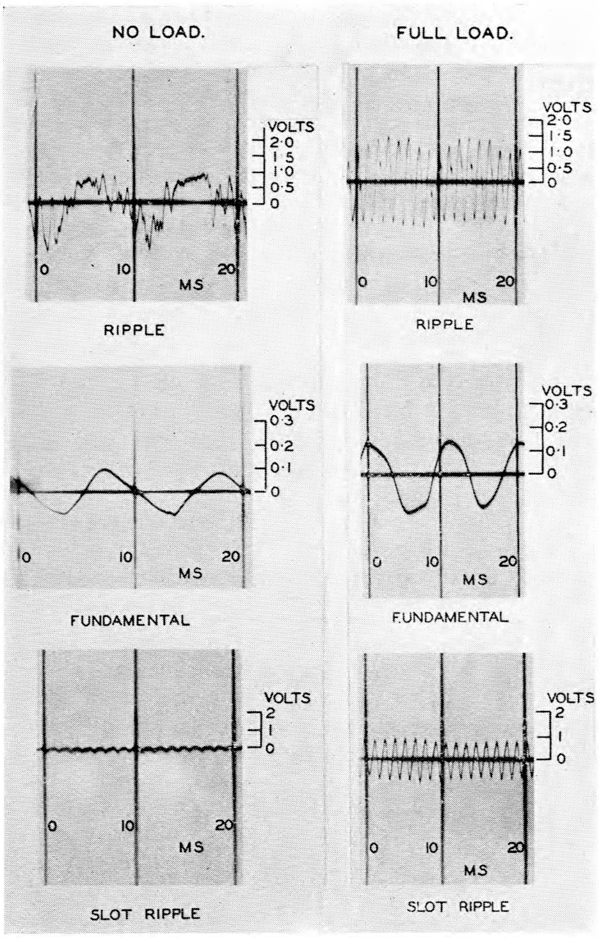

FIG. **4**.—OSCILLOGRAMS FROM D.C. GENERATOR.

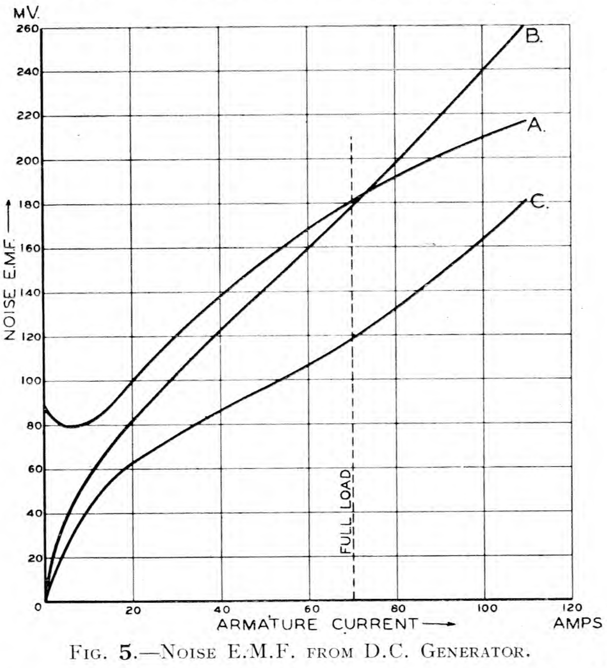

FIG. **5**.—NOISE E.M.F. FROM D.C. GENERATOR.

have been plotted in curve B of Fig. **5**. It will be seen that this curve is almost identical with curve A, which gives the noise E.M.F. from the machine when operating normally. Therefore, slot ripple is almost entirely due to current in the armature. This is in agreement with the results obtained in Appendix I where an attempt has been made to derive, from theoretical considerations, the magnitude of the slot ripple E.M.F. generated in the armature of D.C. machines. Incidentally the numerous factors governing the magnitude have been demonstrated.

Referring to the conclusions arrived at in this Appendix and to Fig. **5** it will be noticed that:—

(i) The slot ripple E.M.F. generated in the armature of a D.C. machine varies with load current and is almost directly proportional to it from a small load up to full load.

(ii) At full load the magnitude of this E.M.F. is approximately 1.5% of the D.C. voltage.

(iii) The magnitude of the noise E.M.F. at full load can be taken as 1% of the D.C. voltage.

Further it will be realised that in order to reduce the slot ripple E.M.F.

(*a*) the reluctance of the magnetic path per slot should be large, and

(*b*) the rate of change of this reluctance should be small.

Greater reluctance can be secured by using open slots and large air gaps between the pole shoes and the armature. The effect of removing the pole shoes on a particular generator is clearly shown by curve C, Fig. **5**. However, large air gaps will reduce the efficiency of the machine.

The change in reluctance can be reduced by several methods:—

(i) Completely enclosed slots—*i.e.*, a smooth core armature. Early D.C. generators for telephone purposes were of this type and little trouble from noise was experienced. However, they were of non-standard type and therefore costly. In addition, the D.C. efficiency was not high and, since this construction increases the inductance of the armature, commutation was difficult. Special brushes and a large number of commutator segments were necessary in order to reduce sparking.

It has been suggested to one manufacturer that an approximation to completely enclosed slots could be obtained if a " stalloy " strip were associated with the slot wedge on a standard design of armature.

(ii) Skewed slots or skewed pole shoes.

(iii) Chamfered pole shoes.

The above modifications will also reduce the E.M.F. induced in the polar windings.

Any concentration of magnetic flux will cause an increase in the rate of change of reluctance. Hence, interpolar windings and interpole shoes should be carefully designed to counteract flux concentration. This will also result in a reduction of noise due to commutation.

If too high a flux density occurs in the armature teeth, the noise E.M.F. will be increased in magnitude on account of intermodulation products.

In spite of the above, it must be realised that the reduction in interference from slot ripple obtained by careful design is small in comparison with that secured

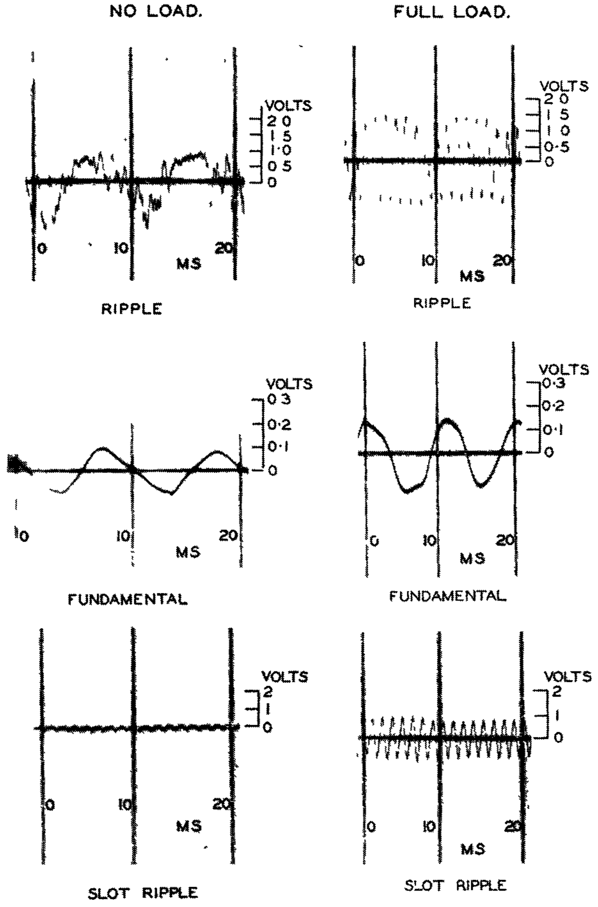

Fig. 4.—Oscillograms from D.C. Generator.

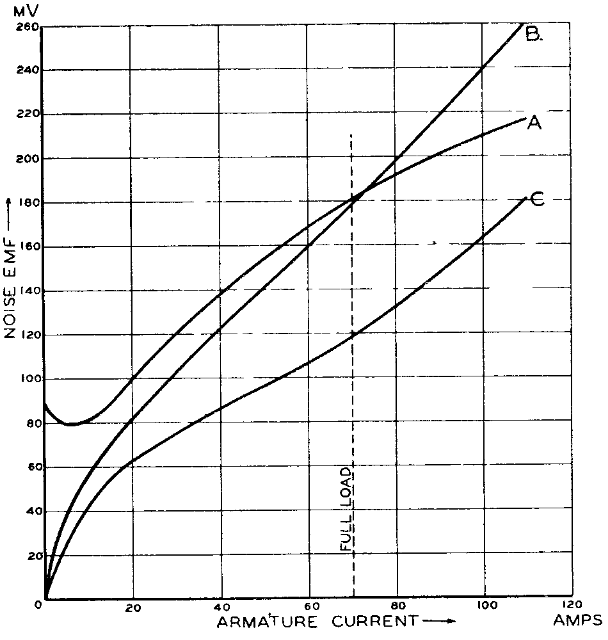

Fig. 5.—Noise E.M.F. from D.C. Generator.

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If too high a flux density occurs in the armature teeth, the noise E.M.F. will be increased in magnitude on account of intermodulation products.

In spite of the above, it must be realised that the reduction in interference from slot ripple obtained by careful design is small in comparison with that secured by a low-pass filter. Hence, in general, a high D.C. efficiency should be aimed at.

(2) Rectifiers[3].

The rectification of a pure sine wave can be expressed by the following series:—

$$y = \frac{m}{\pi} \sin \frac{\pi}{m} \left[1 + 2 \sum_{n=1}^{\infty} \frac{\cos nm\theta}{1 - n^2 m^2} \right]$$

where m is the number of phases.

Hence, if the effective value of the D.C. E.M.F. is known (allowing for voltage drop in the rectifier itself), the approximate value of the noise E.M.F. can be calculated by multiplying the amplitudes at the ripple frequencies by the corresponding weighting factors and obtaining the R.M.S. value of the products. This has been done for the cases of (*a*) Single phase, full wave, and (*b*) Three phase, full wave, rectifiers in Tables 2 and 3, assuming a supply frequency of 50 c/s.

TABLE 2.

(*a*) Single phase, full wave.

Frequency.	% of D.C.	Weighting factor (af).	% × af.
D.C.	100 %	—	—
100 c/s.	47.1	0.015	0.71
200	9.4	0.105	0.99
300	4.04	0.3	1 21
400	2.25	0.4	0.9
500	1.43	0.472	0.67
600	0.99	0.56	0.55
700	0.73	0.705	0.52
800	0.55	1.0	0.55
900	0.44	1.4	0.62
1000	0.35	1.84	0.64
1100	0.29	1.77	0.51
1200	0.25	1.26	0.31
1300	0.21	0.795	0.17
		R.M.S.	2.5%

TABLE 3.

(*b*) Three Phase, Full Wave.

Frequency.	% of D.C.	Weighting Factor (af).	% × af.
D.C.	100	—	—
300 c/s.	4	0.3	1.2
600	0.99	0.56	0.55
900	0.44	1.4	0.62
1200	0.246	1.26	0.31
1500	0.157	0.419	0.066
		R.M.S.	1.5 %

The above values of noise E.M.F. will apply only in the case of a high impedance resistance or inductance load. For other types of load the calculation is very involved.

6. Self Impedance of D.C. Generators and Rectifiers.

(1) D.C. Generators.

The bridge method which was developed in order to measure the impedance of D.C. generators, when running and delivering load, is given in Figs. **6** and **7**.

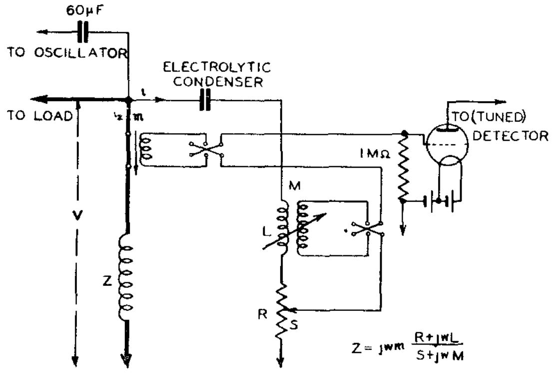

FIG. **6**.—BRIDGE METHOD OF IMPEDANCE MEASUREMENT.

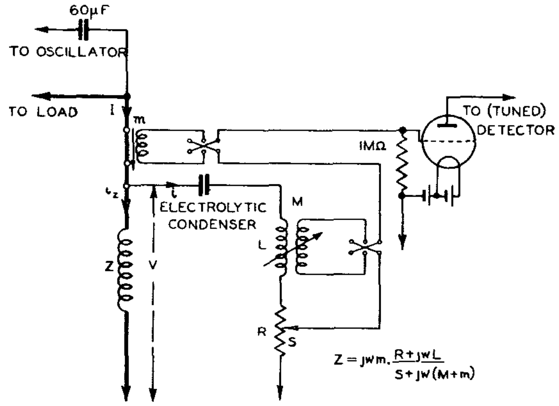

FIG. **7**.—BRIDGE METHOD OF IMPEDANCE MEASUREMENT.

A high resistance, non-reactive potentiometer "R" and the primary of a standard, variable mutual inductance "M" are connected across the output terminals of the generator. The electrolytic condenser eliminates D.C., but must be of high capacitance, and its equivalent series resistance must be low in comparison with "R." The primary of the fixed mutual inductance "*m*" is joined in series with the generator. Details of this are given in Appendix I. The settings "S" and "M" are adjusted until balance is indicated on the tuned detector. The solution of the method is given in Appendix II.

As a result of measurements on a large number of machines it has been found that the impedance of a generator is mainly inductive and that the inductance does not change appreciably under any conditions of running or load. Hence, it is now only necessary to measure the inductance, when the machine is stationary and any normal A.C. bridge can be used. The Owen Bridge given in Fig. **8** has been found to be very convenient for this purpose.

It has been shown elsewhere[1] that the inductance

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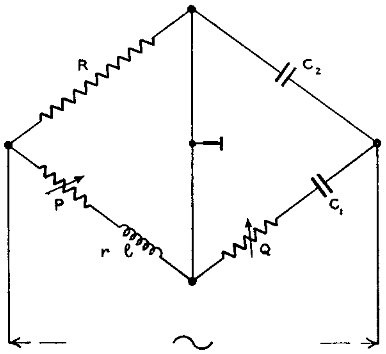
[3] See Bibliography.

[1] See Bibliography.



$$\ell = QRC_2$$

$$r = R\frac{C_2}{C_1} - P$$

FIG. 8.—THE OWEN BRIDGE.

of a D.C. generator is given by

$$L = K.\frac{V}{I.N.} \text{ Henries.}$$

where:—

K is a constant.

V is the rated output voltage of the generator (Volts).

I is the maximum output current of the generator (Amps).

N is the speed of the machine (R.P.M.).

The average value of K has been found to be 1.4 from a large number of measurements.

(2) Rectifiers.

The internal impedance of a rectifier installation is composed of the impedance of equipment connected on the supply side (*e.g.*, leakage impedance of the transformer, etc., anode chokes in the case of mercury arc rectifiers), and the resistance of the rectifier itself. In the case of metal rectifiers the latter can be calculated from the static characteristics of the discs used. Measurements on actual rectifiers have confirmed this. Typical results are given in Table 4.

TABLE 4.

INTERNAL IMPEDANCE OF 24 V. AND 130 V. COPPER OXIDE RECTIFIERS.

Frequency c/s.	Impedance.	
	24 V, 8A, " A " rectifier, type 4-8-16 (A).	130 V, 1A " B " rectifier, type 4-32-2 (A).
150	1.0 /5°	
250	1.1 /9°	
350		42 /7°
450	1.34 /27°	42 /10°
650	1.47 /34°	43.2 /16°
850	1.6 /36°	43.5 /21°
1050	1.8 /38°	43.7 /25°
1250	1.97 /40°	47 /28°
1450	2.16 /43°	
1650	2.39 /46°	52.5 /32°

These values include the voltage regulating resistances, the leakage impedance of the rectifier transformers and the impedance of the leads from the power board to the rectifier rack.

7. Impedance of Batteries.

The actual values of noise P.D. obtained across the load will be practically proportional to the impedance of the battery circuit, since this is generally much lower than that of the load.

From the point of view of impedance to A.C., a battery circuit can be considered as resistance in series with inductance and capacitance.

(1) Resistance.

The total resistance of the battery circuit has two components:—

(i) The resistance of the battery itself, and

(ii) the resistance of connecting leads.

As regards (i) it has been found by actual measurements that the resistance of a secondary cell battery to A.C. is much less than its D.C. resistance, and also changes less with the state of charge. The results of comparative measurements on a typical 6-volt, 20 A*h* battery are given in Fig. **9**. It has been found that the resistance can be taken as $K\frac{N}{Ah}$

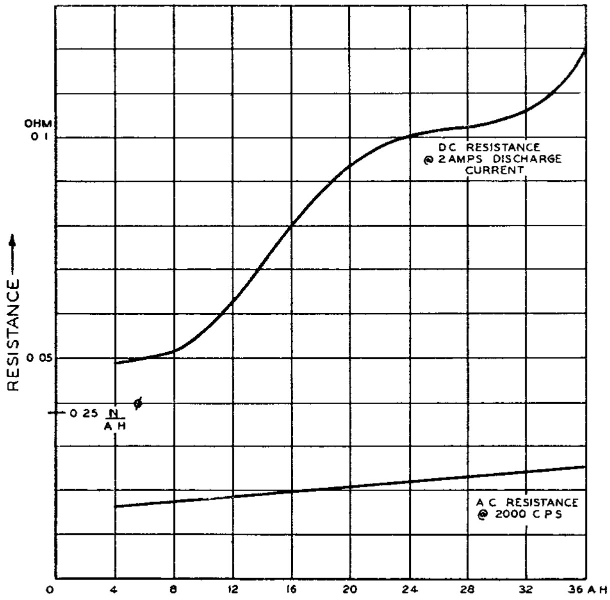


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FIG 9.—A.C. AND D.C. RESISTANCE OF AN ACCUMULATOR DURING DISCHARGE.

where

N is the number of cells of the battery.

A*h* is the capacity of the battery.

K is a constant which can be taken as 0.25.

Tests on particular batteries have given values of K as low as 0.13 but, for the types of cells usually employed in exchanges and repeater stations, 0.25 has been found to be the most reliable value of K.

This value is also quoted by Turner[4] and Vinal[5].

The effective resistance of the leads connecting the battery to the power board will usually be greater than their D.C. resistance on account of skin effect at the noise frequencies.

[4, 5] See Bibliography.

In the case of batteries having a large A*h* capacity, the resistance is negligible in comparison with the reactance.

(2) Inductance.

The inductance of a battery depends on the disposition of the cells and a simple formula which gives satisfactory results is:—

$$L = 4.10^{-9}\left((a + b) \log_e \frac{2ab}{r} - a \log_e (a + d) - b \log_e (b + d)\right) \text{ Henries.}$$

a and b are the lengths of the sides of the rectangle formed by the cells (cms.)

d is the length of the diagonal, *i.e.*, $d = \sqrt{a^2 + b^2}$

r is the radius of the conductor (cms.) (for cells r can be taken as $0.02\ (\mathrm{A}h)^{0.7}$ – A*h* is the capacity of the battery).

The inductance of the leads is given by

$$L = 4.10^{-9}\ l \log_e \frac{d}{r}$$

l is the length of the leads (cms.)

d is the separation (cms.)

r is the radius of each conductor (cms.).

It will be appreciated that for low inductance the separation of the cells and leads should be as small as possible as also should be the length of the connecting leads. Measurements by means of the Owen Bridge have given the following values of overall inductance.

TABLE 5.

Battery Voltage.	Number Tested.	Inductance (μH).		
		MIN.	MAX.	MEAN
24	22	9.5	32.5	17
50	14	9.75	23	18
130	17	22.5	60	38.5

It cannot be emphasised too strongly that any proposed lay-out of a battery should be considered from the point of view of minimum inductance consistent with ease of maintenance.

(3) Capacitance.

The magnitude of the capacitance component is only appreciable in the case of batteries having a small ampere hour capacity and, even then, it is rarely comparable with the resistance. For practical purposes it can generally be neglected.

Results of measurements on a particular small battery indicated that the capacitance could be taken as:—

$$C = 20{,}000 \frac{\mathrm{A}h}{\mathrm{N}} \ (\mu\mathrm{F.}).$$

8. Smoothing the Output from D.C. Generators and Rectifiers.

(1) D.C. Generators.

There are at least four methods of reducing the audio-frequency ripple generated in the armatures of D.C. machines:—

(i) By means of a series choke.
(ii) By a low-pass filter.
(iii) By bridge methods.
(iv) By means of mutual inductance.

(i) When only small direct currents are being considered, a choke is usually adequate to reduce the ripple to a negligible amount. In some cases the inductance of the generator itself may be sufficient. For heavy current machines the size and weight of a suitable choke would be far too unwieldy.

(ii) With the development of electrolytic condensers it has become possible to provide simple low-pass filters for the reduction of interference. A typical filter suitable for a large generator is shown in Fig. **10**.

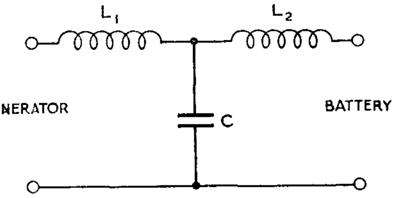

FIG. **10**.—SMOOTHING CIRCUIT (T SECTION).

The general expression for the attenuation ratio of a low-pass filter is given in Appendix V. The principles of design of such a filter are given in Section 9.

(iii) A bridge method has been suggested by Mr. J. A. Sheppard and is the subject of the provisional British Patent Specification No. 403/38. This method was developed after the impedance of a large battery was found to be equivalent to that of an inductive resistance. Fig. **11** gives a diagrammatic sketch of the bridge circuit. The normal conditions of balance obtain in this case, viz., $Z_1Z_4 = Z_2Z_3$. Since the battery inductance is usually only a few microhenries, it will be realised that the balancing chokes required for this method are extremely small and are preferably air cored. To overcome the disadvantage of having two batteries, the modification shown in Fig. **12** was suggested.

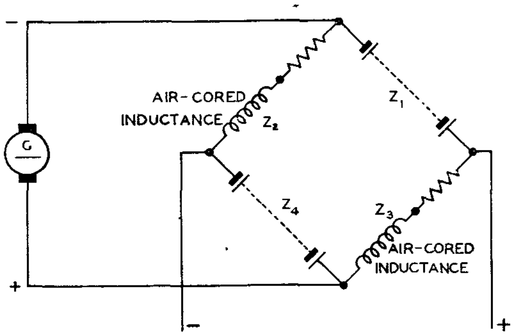

FIG. **11**.—BRIDGE METHOD FOR SMOOTHING THE OUTPUT FROM D.C. GENERATORS.

The modified bridge circuit is satisfactory if the frequencies to be eliminated are high. The method has been tried experimentally and the results are very promising.

It will be appreciated that the cost of a simple low-pass filter for heavy current machines will be very

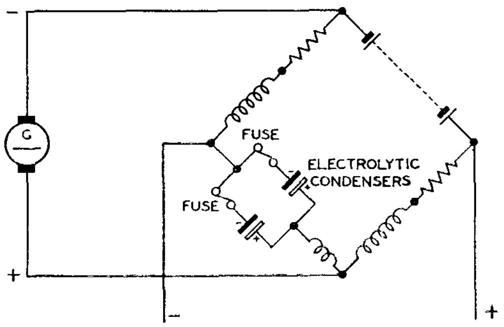

FIG. 12.—BRIDGE METHOD FOR SMOOTHING THE OUTPUT FROM D.C. GENERATORS.

heavy. The proposed scheme would therefore effect a tremendous saving in such cases.

(iv) This method was suggested by Mr. R. O. Carter. With reference to Fig. **13**

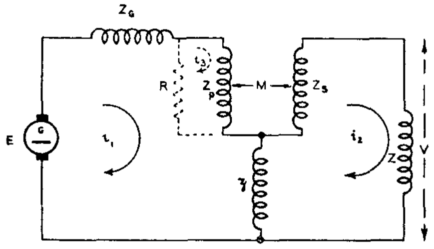

FIG. 13.—MUTUAL INDUCTANCE METHOD FOR SMOOTHING THE OUTPUT FROM D.C. GENERATOR.

$$Mi_1 = (Z + Z_S + z)\, i_2 - z i_1$$

$$i_2 = \frac{M + z}{Z + Z_S + z}\, i_1$$

$$E + Mi_2 = (Z_g + Z_p + z)\, i_1 - z i_2$$

$$E = \left(\frac{(Z_g + Z_p + z)\,(Z + Z_S + z)}{M + z} - (M + z) \right) i_2$$

Now, $V = Zi_2$

$$= Z \cdot \frac{E}{\dfrac{(Z_g + Z_p + z)\,(Z + Z_S + z)}{M + z} - (M + z)}$$

$$= Z \cdot \frac{(M + z)\, E}{(Z_g + Z_p + z)\,(Z + Z_S + z) - (M + z)^2}$$

Hence, $V = 0$, if $M + z = 0$

i.e., $M = -z$

If the resistance of a battery can be neglected then

$$z = r + jwl \div jwl$$

$$\therefore \quad jwM = -jwl$$

$$\text{or } l = -M$$

This scheme has not been given a practical trial, but should be encouraged. The mutual inductance "M" could be quite small and air cored. When large capacity batteries are used their effective resistance is small and the method should be very satisfactory.

(2) Rectifiers.

The smoothing often used for heavy current multiphase mercury arc rectifiers consists of an air cored choke in series with the output from the rectifier, followed by shunt resonant circuits tuned to the various harmonic frequencies. The arrangement is shown in Fig. **14**.

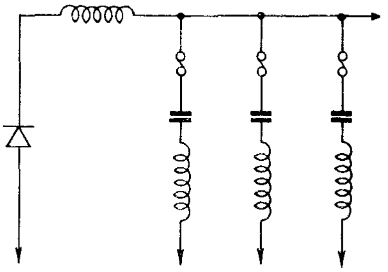

FIG. **14**.—SMOOTHING CIRCUIT FOR RECTIFIERS.

Since the internal impedance of a rectifier is low, particularly at full load, a choke must always be connected directly in series with the output. The performance of the tuned circuits is not entirely satisfactory, and Fig. **15** indicates the effect of frequency drift on the efficiency of the smoothing.

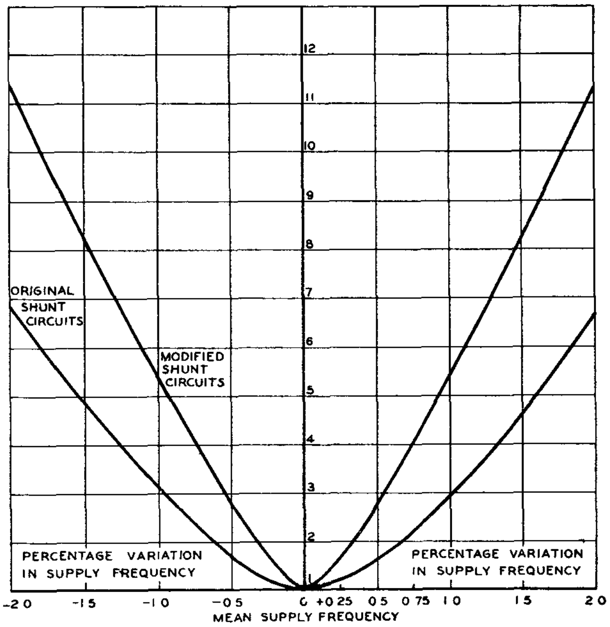

FIG. **15**.—FACTOR BY WHICH NOISE INDUCED FROM RECTIFIERS TO JUNCTIONS MUST BE MULTIPLIED FOR CHANGES IN SUPPLY FREQUENCY.

Again, resonant shunts would be impracticable for smoothing the output from single and three phase rectifiers owing to the number of prominent ripples present. Any of the methods described for D.C. generators should therefore be adopted.

9. Design of smoothing circuits of the Low-pass Filter type.

(1) Data required.

The data required for the design of a smoothing circuit of the low-pass filter type for floating battery equipment is:—

(i) The noise E.M.F. of the generator or rectifier.
(ii) The frequency of this noise E.M.F.
(iii) The self-impedance of the generator or rectifier.
(iv) The impedance of the battery circuit.
(v) The permissible limit of noise P.D. across the battery.

The above may be calculated, measured or assumed from previous experience on similar installations.

For the purpose of designing smoothing equipment, the noise E.M.F. of a generator may be taken as 1% of the D.C. voltage and as occurring at the slot ripple frequency (see Table 1). In the case of rectifiers, the noise E.M.F. at the main ripple frequency is required, since the attenuation of a prototype low-pass filter at the higher frequencies will be considerable (the frequency of this ripple would be 100 c/s for a single phase and 300 c/s for a three phase circuit assuming full wave rectification of a 50 c/s supply). To calculate the magnitude of this ripple from Tables 2 and 3, the D.C. E.M.F. must be known. However, it has been found from experience that the noise E.M.F. at the main ripple frequency can be taken as:—

1.2% of the D.C. output voltage for a single phase, full wave rectifier, and

1.5% of the D.C. output voltage for a three phase, full wave rectifier.

To obtain the self impedance of a D.C. generator, the formula given in Section 6 (1) $\left(L = \frac{1.4\ V}{I.N.} \right)$ can be used. The resistance component can be neglected.

In the case of rectifiers, the self impedance can be neglected in comparison with the input impedance of an adequate smoothing filter.

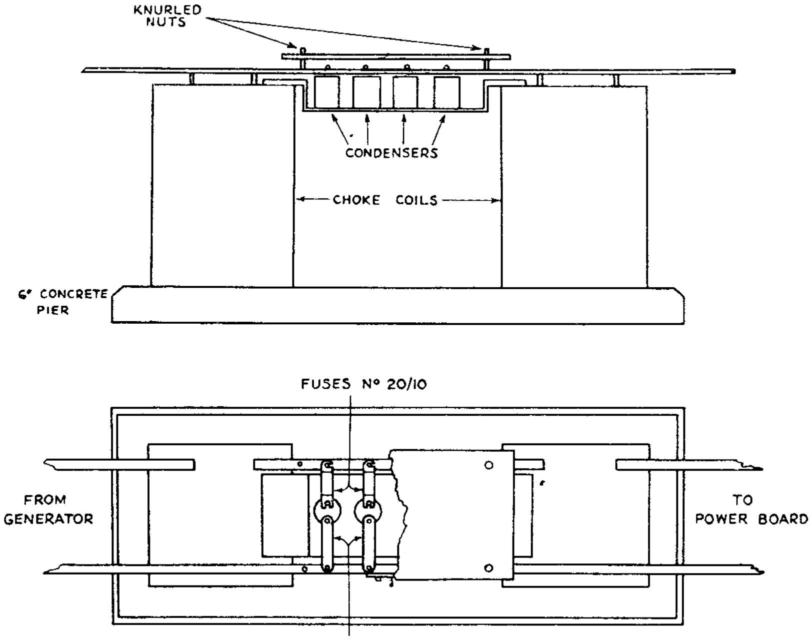

FIG. **16**—50 V. & 60 V. AUTOMATIC EXCHANGES. DIVIDED BATTERY FLOAT POWER PLANTS. TYPICAL ARRANGEMENT OF FILTER EQUIPMENT.

If the impedance of the battery circuit cannot be measured, the inductance may be assumed to have the average value given in Table 5 or it can be calculated as described in Section 7. The resistance can be calculated from the empirical formula given in that Section.

The usual limits of noise P.D. across the discharge leads are given in Section 3.

(2) Arrangement of the Filter.

In order that the maximum smoothing effect shall be obtained from the filter components their arrangement must be carefully chosen. Leads carrying alternating current in opposite directions should be run as close together as possible in order to reduce the external field produced. The inductive impedance of condenser leads will be reduced by this method. The length and resistance of condenser leads should be as small as possible and the rating of fuses used with condensers should be as high as possible.

The ideal lay-out of a filter unit is that in which the generator positive and negative leads are brought to the unit, and the smoothed output is taken from

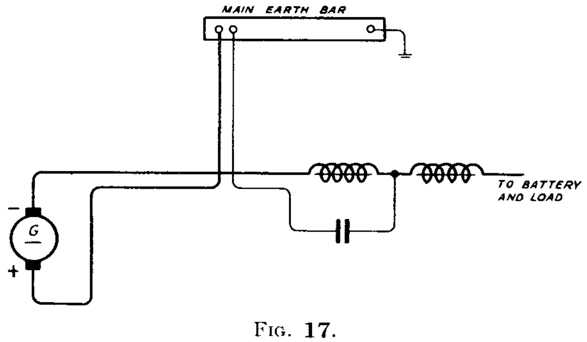

FIG. **17**.

the filter by a separate pair of leads. This arrangement is illustrated in Fig. **16**. Where this lay-out cannot be employed, a satisfactory filter can be constructed, provided that the principle of pairing leads carrying approximately equal alternating current in opposite directions is maintained. This principle is illustrated in Fig. **17**.

When it is not possible to make use of the ideal filter lay-out, a capacitance of 4,000 μF. is the maximum which can usefully be employed. Each condenser bank should be divided into at least two separately fused units. Furthermore, the capacitance of unit should not exceed 2,000 μF.

(3) Design Factor.

In order to offset any inaccuracy in the data employed, and bearing in mind the assumptions made, it is necessary, when designing a smoothing filter, to allow a design factor or ratio between the calculated attenuation and the actual value. The factor taken should be between 1.5 and 4, depending on (*a*) the accuracy with which the quantities involved are known and (*b*) the total cost of the filter.

(4) Design of the Filter.

Knowing the generator E.M.F., the permissible limit of noise P.D. across the battery and the design factor, the required attenuation of the filter can be calculated. The relative values of capacitance and inductance in the filter will depend upon economic considerations; for minimum total cost of the filter, the cost of each choke should be approximately equal to the first cost plus the present value of the annual charges necessary to maintain the capacitance of each condenser bank during the life of the plant.

The simplest method of designing a filter is to try various reasonable values of inductance and capacitance and then to calculate the attenuation ratio at the required frequency by means of the approximate formula given in Appendix V. Little advantage will be obtained by using different sizes of chokes and condenser banks in a multi-stage filter.

Care should be taken to ensure that the resonance frequency $\left(f_r = \frac{1}{2\pi}\sqrt{\frac{2}{LC}} \right)$ is less than 200 c/s, in the case of D.C. generators—in order to provide adequate attenuation for any large components in the generator noise E.M.F. at frequencies less than the slot ripple—or the lowest ripple frequency in the case of rectifiers.

In general, the larger the number of stages in a filter designed to give a certain attenuation, the smaller the total inductance and capacitance. However, the resonance frequency will increase with the number of sections and will set an upper limit.

Acknowledgments.

The authors desire to express their thanks to the Engineer-in-Chief for giving his consent to the reading of a paper which includes a considerable amount of official information. They also wish to record their appreciation of the assistance which they have received from the following manufacturers:—Messrs. Austinlite; Messrs. Bruce Peebles; Messrs. Crompton Parkinson; The Electric Construction Co.; The General Electric Co.; The Hackbridge Electric Construction Co.; Messrs. Newton Bros.; The Power Equipment Co.; Messrs. Standard Telephones & Cables; and The Zenith Electric Co.

Finally, the authors acknowledge their indebtedness to all their colleagues in the Engineering Department who have co-operated in the work described in the paper and particularly to Mr. C. B. Orton for his valuable assistance and suggestions during the earlier tests.

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APPENDIX I.

Magnitude of Slot ripple from the armature of a D.C. Generator.

Let

N = speed of rotation in r.p.m. ;
Z = total number of armature conductors ;
B_g = flux density in air gap ;
D = diameter of armature in cm. ;
l = length of armature in cm. ;
$\gamma = \dfrac{\text{polar arc}}{\text{pole pitch}}$;
c = number of circuits in parallel between brushes ;
x = number of slots in the armature ;
R = reluctance of magnetic path/slot ;
e = induced e.m.f./slot ;
E = total slot ripple E.M.F. generated in the armature.

Then, Conductors/slot $= \dfrac{Z}{x}$

Conductors in series/slot $= \dfrac{Z}{cx}$

Direct Current/conductor $= \dfrac{I}{c}$

The flux produced/slot

$$\phi = \frac{\frac{4\pi}{10}\cdot\frac{I}{c}\cdot\frac{Z}{x}}{R}$$

$$\therefore\ e = \frac{Z}{2cx}\cdot\frac{d\phi}{dt}\cdot 10^{-8}\ \text{Volts}$$

$$= \frac{Z}{2cx}\cdot\frac{d\phi}{dR}\cdot\frac{dR}{dt}\cdot 10^{-8}\ \text{Volts}$$

$$= -\frac{Z}{2cx}\cdot\frac{\frac{4\pi}{10}\cdot\frac{I}{c}\cdot\frac{Z}{x}}{R^2}\cdot\frac{dR}{dt}\cdot 10^{-8}\ \text{Volts. ..(1)}$$

But $R \propto \dfrac{1}{l}$

and $\dfrac{dR}{dt} \propto \dfrac{1}{l}\cdot Nx$

Hence, $\dfrac{1}{R^2}\cdot\dfrac{dR}{dt} \propto Nxl$

$$\therefore\ e \propto \frac{Z^2}{cx}\cdot\frac{I}{c}\cdot Nl \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

But the full-load current from a D.C. generator can be shown to be

$$I = \frac{\pi D r_s t f_c}{Z}\Delta\cdot c$$

where $r_s = \dfrac{\text{slot width}}{\text{slot pitch}}$
t = depth of slot
$f_c = \dfrac{\text{net copper per slot}}{\text{slot area}}$ (copper space factor)
Δ = current density in copper.

Thus, $\dfrac{I}{c} \propto \dfrac{D}{Z}$

$$\therefore\ e \propto \frac{Z}{cx}\cdot DNl$$

The total E.M.F. induced in the armature will probably be the R.M.S. sum of the individual E.M.F.'s per slot, viz.:—

$$E = \sqrt{e_1^2 + e_2^2 + e_3^2 + \ \ldots\ldots e^2_x}$$

$$\propto e\sqrt{x}$$

$$= \frac{Z}{c}DNl\cdot\frac{1}{\sqrt{x}}.$$

Now the E.M.F. developed in the armature of a D.C. generator is given by

$$V = \frac{N}{60}ZB_g\pi Dl\frac{\gamma}{c}\cdot 10^{-8}$$

$$\therefore\ E \propto \frac{V}{\sqrt{x}}$$

Thus, it would appear that the slot ripple E.M.F. generated in the armature of a D.C. machine is independent of the actual full load current, but is directly proportional to the D.C. voltage. The results of measurements carried out on a large number of machines are summarised in Tables 6, 7, 8, 9, 10 and 11.

It must be remembered that the machines, details of which are given in these tables, are of different types, supplied by different contractors and of different outputs. Hence, it is rather surprising that the values of $E\sqrt{x}$ do not show a wider variation.

Conclusions.

(1) It has been found in practice that the mean value of the slot ripple E.M.F. from a D.C. generator is approximately 1.5% of the D.C. voltage.

(2) When designing smoothing circuits the magnitude of the noise E.M.F. can be taken as 1% of the D.C. voltage of the machine.

(3) From equation (2) above, it will be apparent that the magnitude of the slot ripple E.M.F. generated in the armature of a particular machine should be directly proportional to the direct current in the armature. This has been demonstrated by actual measurements (**see Section 5 (1), Fig. 5**).

(4) From equation (1)
  (*a*) the larger the reluctance of the magnetic path per slot, and
  (*b*) the smaller the change in this reluctance, the smaller will be the induced E.M.F.

TABLE 6.

NOISE AND RIPPLE E.M.F. FROM D.C. GENERATORS AT FULL LOAD.
1. Repeater Station " A " generators.
a. Earlier types.

D.C. Output.		Noise E.M.F. E^1 (mV.)	$\frac{E^1}{V}$ X100	Speed N (R.P.M.)	Armature Slots x	Slot-ripple frequency $\frac{N.\,x}{60}$ c/s	Slot-ripple E.M.F. E (mV.)	$\frac{E}{V}$ X100	$\frac{E}{V}\sqrt{x}$
Volts V.	Amps I.								
25	200	80	0.32	950	100	1580	191	0.77	76.4
25	250	118	0.47	1285	63	1350	193	0.77	61.2
25	300	354	1.42	935	72	1120	212	0.85	54.8
25	300	197	0.79	935	72	1120	118	0.47	30.6
Mean			0.75%	—	—	—	—	0.71%	55.7

TABLE 7.

NOISE AND RIPPLE E.M.F. FROM D.C. GENERATORS AT FULL LOAD.
1. Repeater Station " A " generators.
b. New machines.

D.C. Output.		Noise E.M.F. E^1 (mV.)	$\frac{E^1}{V}$ X100	Speed N (R.P.M.)	Armature Slots x	Slot-ripple frequency $\frac{N\,x}{60}$ c/s	Slot-ripple E.M.F. E (mV.)	$\frac{E}{V}$ X100	$\frac{E}{V}\sqrt{x}$
Volts V.	Amps I.								
25	60	246	0.97	1440	19	456	560	2.24	97.6
25	60	198	0.79	1440	19	456	450	1.8	78.4
25	75	200	0.80	1450	32	770	222	0.89	50.0
25	75	306	1.22	1450	32	770	340	1.36	76.8
25	75	266	1.06	1450	32	770	296	1.18	66.8
25	75	247	0.97	1450	32	770	276	1.10	62.4
25	120	124	0.50	1500	57	1425	248	0.99	74.8
25	120	265	1.06	1500	57	1425	530	2.12	160.0
25	120	81	0.32	1500	57	1425	163	0.65	49.2
25	120	264	1.05	1500	57	1425	528	2.11	159.2
25	120	201	0.80	1500	57	1425	402	1.6	121.6
25	120	288	1.15	1500	57	1425	576	2.3	174.0
25	150	229	0.92	950	35	555	444	1.76	105.2
25	150	194	0.78	950	35	555	376	1.50	88.8
25	200	188	0.75	950	40	632	313	1.25	79.2
25	300	196	0.78	950	40	632	318	1.27	80.4
25	300	176	0.70	950	40	632	286	1.14	72.4
25	300	164	0.66	950	40	632	266	1.06	67.2
25	300	157	0.63	950	40	632	255	1.02	64.4
25	300	255	1.02	950	40	632	414	1.66	104.4
25	300	282	1.13	950	40	632	458	1.83	116.0
25	600	247	0.99	1000	64	1065	132	0.53	42.0
Mean (*a*)			0.86%					1.42%	90.5
Standard deviation (σ)			0.22%					0.50%	28.8
$\frac{\sigma}{a}$			0.26					0.35	0.32

TABLE 8.

NOISE AND RIPPLE E.M.F. FROM D.C. GENERATORS AT FULL LOAD.
2. Repeater Station " B " generators.
a. Earlier types.

D.C. Output.		Noise E.M.F. E^1 (mV.)	$\frac{E^1}{V}$ X100	Speed N (R.P.M.)	Armature Slots x	Slot-ripple frequency $\frac{N.\,x}{60}$ c/s	Slot-ripple E.M.F. E (mV.)	$\frac{E}{V}$ X100	$\frac{E}{V}\sqrt{x}$
Volts V.	Amps I.								
130	15	1170	0.90	930	29	450	2660	2.05	110
130	15	815	0.63	930	29	450	1850	1.42	77
130	25	4370	3.36	1200	26	520	8930	6.87	350
130	25	2080	1.60	1200	26	520	4950	3.81	194
130	50	1430	1.10	1000	74	1230	1190	0.79	68
Mean			1.52%					2.99%	160

TABLE 9.

NOISE AND RIPPLE E.M.F. FROM D.C. GENERATORS AT FULL LOAD.

2. Repeater Station " B " generators.

b. New machines.

D.C. Output.		Noise E.M.F. E^1 (mV.)	$\frac{E^1}{V}$ X100	Speed N (R.P.M.)	Armature Slots x	Slot-ripple frequency $N.\frac{x}{60}$ c/s	Slot-ripple E.M.F. E (mV)	$\frac{E}{V}$ X100	$\frac{E}{V}\sqrt{x}$
Volts V.	Amps I.								
130	15	1820	1.40	1440	19	455	4040	3.11	135
130	15	1780	1 37	1440	19	455	3960	3.05	132
130	15	1510	1.16	1440	19	455	3360	2.59	112
130	15	1780	1.37	1440	19	455	3960	3 05	132
130	15	1180	0.91	1440	32	770	1410	1 08	61
130	15	1260	0.97	1440	32	770	1500	1.15	65
130	15	1800	1.38	1440	32	770	2030	1.56	88
130	15	1040	0.80	1440	32	770	1240	0.95	54
150	15	880	0.68	710	47	555	1720	1.15	79
150	15	1010	0.78	950	35	555	1980	1 32	78
150	15	2020	1.55	950	36	570	3740	2 49	149
130	30	880	0.68	950	35	555	1720	1.32	78
130	30	1100	0.85	950	35	555	2160	1.66	99
130	40	670	0.52	950	35	555	1320	1.01	60
130	40	480	0.37	950	35	555	930	0.72	42
130	40	800	0.62	950	35	555	1560	1 20	71
130	40	750	0.58	950	35	555	1480	1 14	67
130	45	370	0.29	1000	49	820	360	0.28	19
Mean (*a*)			0.91%					1.60%	84
Standard deviation (σ)			0.37%					0.84%	34
σ/a			0.41					0.52	0.41

TABLE 10.

NOISE AND RIPPLE E.M.F. FROM D.C. GENERATORS AT FULL LOAD.

3. Exchange generators.

a. Earlier types.

D.C. Output.		Noise E.M.F. E^1 (mV.)	$\frac{E^1}{V}$ X100	Speed N (R.P.M.)	Armature Slots x	Slot-ripple frequency $N\frac{x}{60}$ c/s	Slot-ripple E.M.F. E (mV.)	$\frac{E}{V}$ X100	$\frac{E}{V}\sqrt{x}$
Volts V.	Amps I.								
57	110	273	0.48	1000	50	833	241	0.42	29.9
50	180	1250	2.50	950	40	634	2050	4.10	259
50	180	1130	2.26	950	40	634	1860	3.73	235
60	190	1440	2 40	960	40	640	2360	3 93	249
60	190	1360	2 27	960	40	640	2230	3.73	235
50	220	344	0.69	960	42	672	506	1.01	65 6
50	220	328	0.66	960	42	672	497	0 99	64.5
60	280	415	0.69	1000	132	2200	1840	3.07	352
60	400	1750	2.92	800	52	695	2500	4.16	301
60	400	805	1 34	800	52	695	1150	1.92	138
50	480	2170	4.35	1000	48	800	2170	4.35	300
57	800	730	1.28	750	75	937	471	0 83	71.6
69	850	1060	1.54	590	92	900	757	1 10	93.0
50	900	246	0 49	600	180	1800	852	1 70	229
69	1000	480	0.70	750	68	850	400	0 58	47.7
50	1200	533	1 06	600	94	940	338	0 68	65.6
50	1200	730	1 46	600	94	940	462	0 93	89.8
57	1200	850	1 49	600	110	1100	480	0 84	87.5
50	1600	137	0.27	600	80	800	137	0.27	24.0
50	1600	270	0 54	600	179	1790	910	1.82	243
50	1600	380	0.76	600	179	1790	1280	2.56	342
50	1600	375	0 75	600	179	1790	1260	2 52	337
50	1600	230	0 46	600	179	1790	775	1 55	207
50	1600	720	1.44	600	180	1800	2490	4 98	670
50	1600	620	1 24	600	180	1800	2150	4.30	578
50	1825	1200	2.40	600	102	1020	656	1.29	133
50	1825	590	1.18	400	108	720	777	1.55	161
Mean			1.39%					2.18%	208

TABLE 11.

NOISE AND RIPPLE E.M.F. FROM D.C. GENERATORS AT FULL LOAD.

3. Exchange generators.

b. New machines.

D.C. Output. Volts V.	D.C. Output. Amps I	Noise E.M.F. E^1 (mV.)	$\frac{E^1}{V}$ X100	Speed N (R.P.M.)	Armature Slots λ	Slot-ripple frequency $\frac{N.\lambda}{60}$ c/s	Slot-ripple E.M.F. E (mV.)	$\frac{E}{V}$ X100	$\frac{E}{V}\sqrt{\lambda}$
50	100	490	0.98	950	36	570	920	1.84	110
50	100	420	0.84	950	36	570	790	1.58	94.8
50	100	550	1.10	950	36	570	1030	2.06	123
50	100	400	0.80	950	36	570	753	1.51	90.3
50	100	425	0.85	950	36	570	800	1.60	96.0
50	200	316	0.63	950	37	585	574	1.15	69.7
50	200	320	0.64	950	37	585	582	1.16	70.7
50	200	310	0.62	950	37	585	563	1.13	68.5
50	300	520	1.04	950	32	507	1080	2.16	122
50	300	630	1.26	950	32	507	1310	2.62	148
50	300	680	1.36	950	32	507	1420	2.83	160
50	400	490	0.98	960	52	833	433	0.87	62.5
50	400	680	1.36	960	52	833	602	1.20	87.0
50	800	262	0.52	730	56	680	391	0.78	58.5
Mean (a)			0.92%					1.61%	97
Standard deviation (σ)			0.29%					0.6%	31
σ/a			0.32					0.37	0.32

APPENDIX II.

Design of Mutual Inductance.

Fig. **18** shows a suitable robust form of construction for the mutual inductance. The primary is conveniently a copper or brass rod of sufficient cross-sectional area to carry the maximum D.C. required. An earthed screen completely surrounds the primary, but is insulated from it. The secondary consists of a toroidal winding having a large number of turns wound on a split wooden core secured as close as possible to the primary.

The design is influenced by the following considerations. With reference to Fig. **19**

$$\text{Flux enclosed by core} = \phi = \mu l \int_{\left(a - \frac{t}{2}\right)}^{\left(a + \frac{t}{2}\right)} H\,dr.$$

where μ = permeability of the medium

$$\phi = \mu l \int_{\left(a - \frac{t}{2}\right)}^{\left(a + \frac{t}{2}\right)} \frac{2}{10}\,\frac{I}{r}\,dr$$

$$= \frac{2}{10}\,\mu l\,I \log.\frac{1 + \frac{t}{2a}}{1 - \frac{t}{2a}}$$

If $\frac{t}{2a}$ is small, then

$$\log_e \frac{1 + \frac{t}{2a}}{1 - \frac{t}{2a}} = \left(\frac{t}{2a} - \frac{1}{2}\left(\frac{t}{2a}\right)^2 + \text{etc.}\right)$$

$$- \left(- \frac{t}{2a} - \frac{1}{2}\left(\frac{t}{2a}\right)^2 - \text{etc.}\right)$$

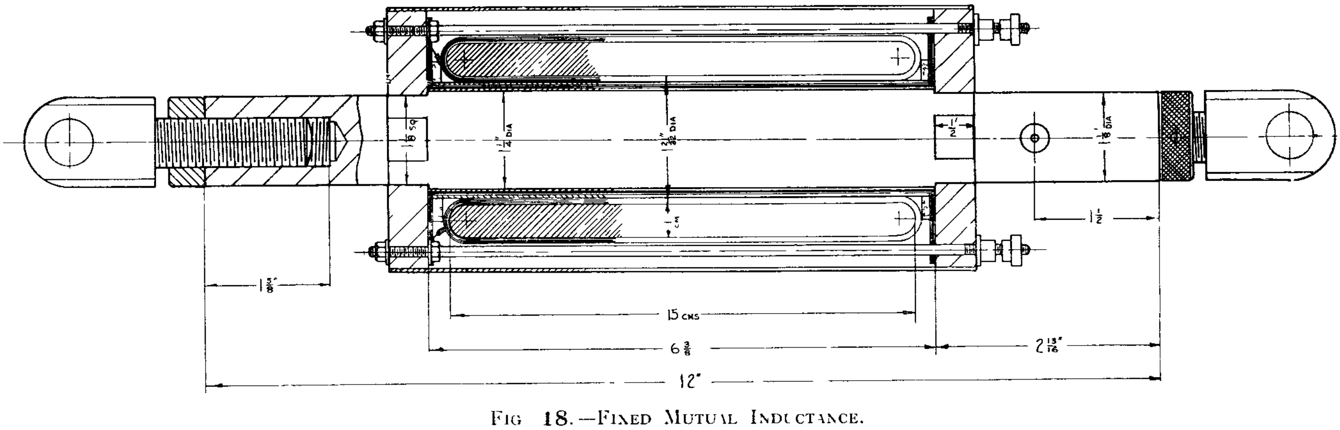


FIG. 18.—FIXED MUTUAL INDUCTANCE.

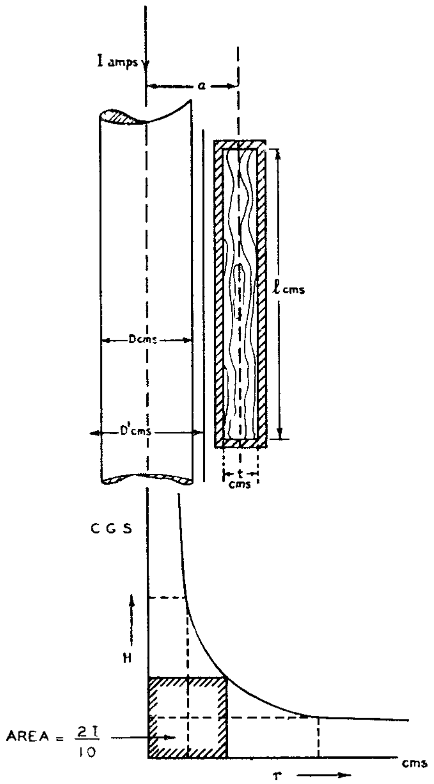


Fig. 19

$$\doteqdot \frac{t}{a}$$

$$\therefore \quad \phi \doteqdot \frac{2}{10a}\,\mu l t I$$

Mutual inductance:—

$$m = \phi\frac{T}{I}\cdot 10^{-8} = \frac{2}{10a}\,\mu l t T\cdot 10^{-8}\text{ Henries}$$

For m to be large a should be small and l as great as possible. If t is increased substantially, the above formula will not apply and the increase in mutual inductance will not be proportional to t.

Again, if m is to be constant over a frequency range, μ should be constant. The medium should be air, *i.e.*, $\mu = 1$.

The diameter of the primary bar is given by:—

$$\frac{\pi}{4}D^2 = \frac{I}{1000}$$

e.g. If $T = 5000$
$t = 1$ cm.
$l = 15$ cms.
and $a = 3$ cms.

$$m = \frac{2}{10a}\,ltT\cdot 10^{-2}\,\mu\text{H.} = 50\,\mu\text{H.}$$

The self inductance of the secondary is:—

$$\frac{4\,\frac{\pi}{10}\,IT}{2\pi a}\cdot lt\cdot\frac{T}{I}\cdot 10^{-8}\text{ Henries}$$
$$= Tm.$$

This affords a good check of winding.

Errors introduced.

Capacity between screen and primary

$$= \frac{K}{2\log_e\frac{D_1}{D}}\cdot\frac{1}{9}\cdot 10^{-11}\text{ Farads.}$$

In an actual mutual inductance the value of this capacity was 800 $\mu\mu$F., and this will act as a shunt across the impedance under test. In practice it is necessary to use fairly long leads from the secondary of the mutual inductance, and these will increase the effective self capacity of the winding.

The effect of capacity across the secondary of a mutual inductance is considered below.

With reference to Fig. **20.**

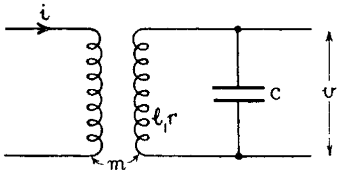


Fig. **20.**

$$v = \frac{1}{j\omega C}\cdot\frac{j\omega m i}{r + j\omega l + \frac{1}{j\omega C}} \qquad \omega = 2\pi f$$

If m^1 is the effective mutual:—

$$j\omega m^1 i = v$$

$$\text{or } m^1 = \frac{m}{1 - \omega^2 lC + j\omega Cr}$$

Thus, as the frequency is increased to

$$\omega = \frac{1}{\sqrt{LC}}$$

m^1 will increase and afterwards decrease.

This is shown by the upper curve of Fig. **21**, which gives the results obtained with a mutual inductance

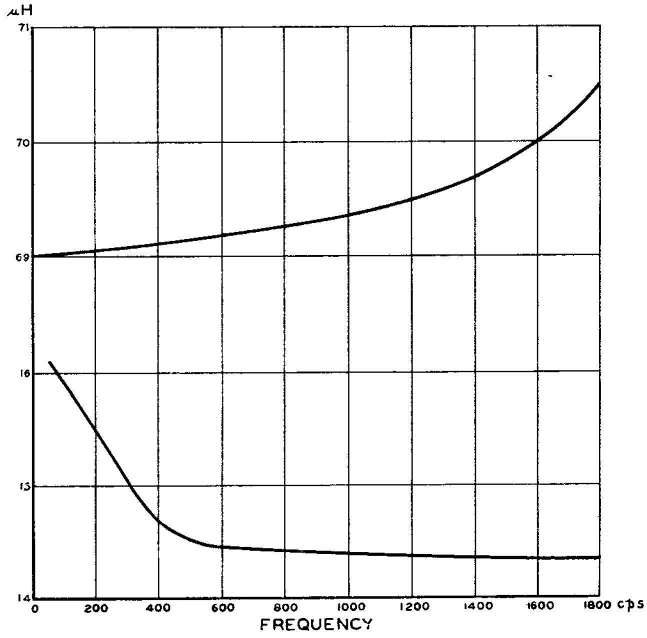


Fig. **21.**—Calibration of Mutual Inductances.

capable of carrying up to 1500 amps. D.C. in the primary bar.

There are other factors which affect the variation of mutual inductance with frequency. For example, with a design in which two flat coils were fixed on opposite faces of a rectangular copper bar, the variation obtained is shown by the lower curve on Fig. **21**. This has been proved to be governed by the variation with frequency of the flux distribution round the rectangular bar.

APPENDIX III.

Solutions for Method of Impedance Measurement.

Referring to

(*a*) Fig. **6** we have:—

$$V = Zi_z = (R + j\omega L)i$$

i.e., $$i_z = \frac{V}{Z} \;\&\; i = \frac{V}{R + j\omega L}$$

At balance:—

$$j\omega m i_z + (S + j\omega M)i = 0$$

or $$j\omega m\frac{V}{Z} + (S + j\omega M)\cdot\frac{V}{R + j\omega L} = 0$$

or $$Z = -\,j\omega m\cdot\frac{R + j\omega L}{S + j\omega M}$$

(*b*) Fig. **7**.

As above:—

$$i_z = \frac{V}{Z} \;\&\; i = \frac{V}{R + j\omega L}$$

$$I = i + i_z = V\left(\frac{1}{Z} + \frac{1}{R + j\omega L}\right)$$

At balance:—

$$j\omega mI + (S + j\omega m)i = 0$$

or $$j\omega m\left(\frac{1}{Z} + \frac{1}{R + j\omega L}\right)V + (S + j\omega M)\cdot\frac{V}{R + j\omega L} = 0$$

$$j\omega m\cdot\frac{1}{Z} + \frac{S + j\omega(M + m)}{R + j\omega L} = 0$$

$$\&\quad Z = -\,j\omega m\frac{R + j\omega L}{S + j\omega(M + m)}$$

This method is very useful for comparing mutual inductances. For, if Z is known, we have:—

$$ZS + j\omega(M + m)Z = -\,j\omega mR + \omega mL$$

Equating the unreal terms

$$MZ + mZ = -\,mR$$

or $$m = -\frac{Z}{Z + R}M$$

The negative sign indicates that *m* must be reversed for balance.

The vector diagrams in Fig. **22** illustrate the balance conditions of the two methods for the cases:—

(*a*) When the angle ϕ, of the impedance under test, is greater than $\tan^{-1}\frac{\omega L}{R}$

and (*b*) $\phi < \tan^{-1}\frac{\omega L}{R}$

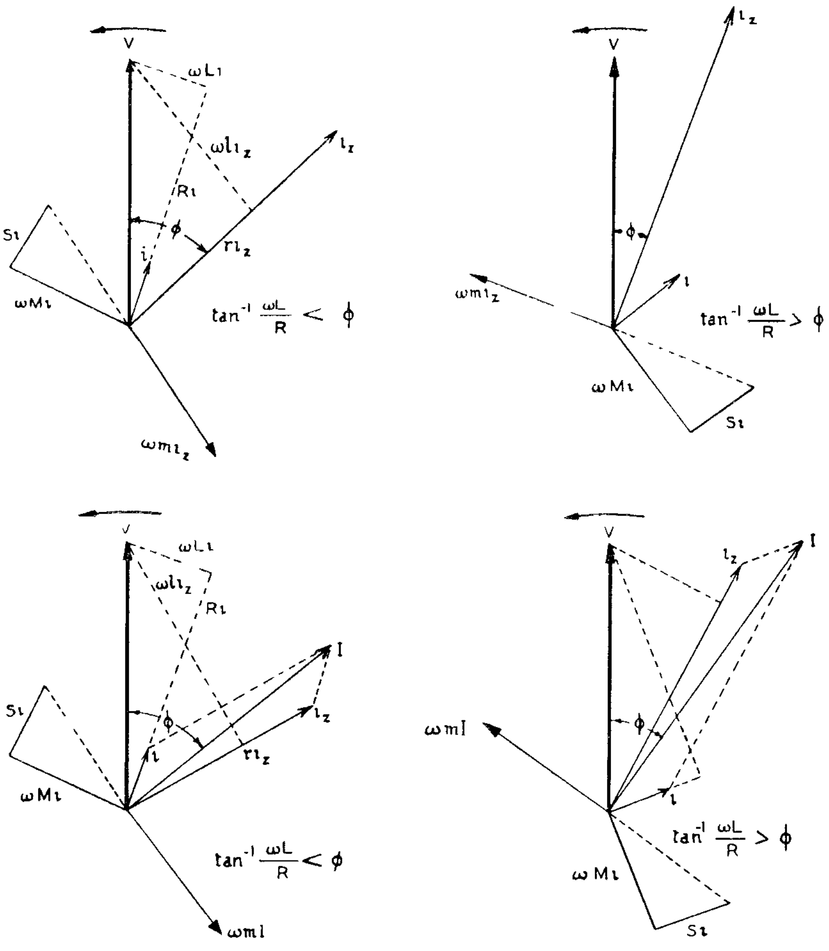


FIG. 22.—VECTOR DIAGRAMS.

APPENDIX IV.

Solution of 4-stage network.

Referring to Fig. **23**

$$Z_G\, i_1 + Z_6\, (i_1 - i_2) = E$$
$$Z_1 i_2 + Z_5\, (i_2 - i_3) + Z_6\, (i_2 - i_1) = 0$$
$$Z_2\, i_3 + Z_4\, (i_3 - i_4) + Z_5\, (i_3 - i_2) = 0$$
$$Z_3 i_4 + Z_B i_4 + Z_4\, (i_4 - i_3) = 0$$

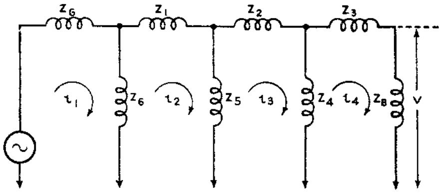


Fig. 23.

Rewriting: —

$$(Z_G + Z_6)\, i_1 = E + Z_6\, i_2$$
$$(Z_1 + Z_5 + Z_6)\, i_2 - Z_5\, i_3 - Z_6\, i_1 = 0$$
$$(Z_2 + Z_4 + Z_5)\, i_3 - Z_4 i_4 - Z_5\, i_2 = 0$$
$$(Z_3 + Z_4)\, i_4 - Z_4\, i_3 + Z_B\, i_4 = 0$$
$$(Z_3 + Z_4 + Z_B)\, i_4 - Z_4\, i_3 = 0$$

Whence,

$$i_1 = \frac{E + Z_6\, i_2}{Z_G + Z_6}$$

$$\therefore \Big((Z_1 + Z_5 + Z_6)\,(Z_G + Z_6) - Z^2{}_6\Big)\,.\, i_2 - Z_5\,(Z_G + Z_6)\, i_3 - Z_6\, E = 0$$

$$\therefore \quad i_2 = \frac{Z_5(Z_G + Z_6)\, i_3 + Z_6\, E}{(Z_1 + Z_5 + Z_6)\,(Z_G + Z_6) - Z^2{}_6}$$

$$\Big[(Z_2 + Z_4 + Z_5)\Big((Z_1 + Z_5 + Z_6)(Z_G + Z_6) - Z^2{}_6\Big) - (Z_G + Z_6)Z^2{}_5\Big]\, i_3$$
$$- Z_4\Big((Z_1 + Z_5 + Z_6)\,(Z_G + Z_6) - Z^2{}_6\Big) i_4 - Z_5 Z_6\, E = 0$$

$$\therefore \quad i_3 = \frac{Z_4\Big(\,(Z_1 + Z_4 + Z_5 + Z_6)\,(Z_G + Z_6) - Z^2{}_6\,\Big) i_4 + Z_5\, Z_6\, E}{(Z_2 + Z_4 + Z_5)\Big(\,(Z_1 + Z_5 + Z_6)\,(Z_G + Z_6) - Z^2{}_6\Big) - (Z_G + Z_6)Z^2{}_5}$$

$$i_4 \quad (Z_3 + Z_4 + Z_B)\Big[(Z_2 + Z_4 + Z_5)\Big((Z_1 + Z_5 + Z_6) \cdot (Z_G + Z_6) - Z^2{}_6\Big) - (Z_G + Z_6)\, Z^2{}_5\Big]$$
$$- Z^2{}_4\Big[(Z_1 + Z_4 + Z_5 + Z_6) \cdot (Z_G + Z_6) - Z^2{}_6\Big] - Z_4\, Z_5\, Z_6\, E = 0$$

But $V = Z_B\, i_4$

$$\therefore \frac{E}{V} = \frac{1}{Z_B\, Z_4\, Z_5\, Z_6} \cdot \Big[(Z_3 + Z_4 + Z_B) \cdot (Z_2 + Z_4 + Z_5)\Big((Z_1 + Z_5 + Z_6) \cdot (Z_G + Z_6) - Z^2{}_6\Big)$$
$$- Z^2{}_4\Big((Z_1 + Z_4 + Z_5 + Z_6) \cdot (Z_G + Z_6) - Z^2{}_6\Big)\Big]$$

Whence

$$\frac{E}{V} = 1 + \frac{Z_G + Z_1 + Z_2 + Z_3}{Z_B} + \frac{Z_G}{Z_6} + \frac{Z_G + Z_1}{Z_5} + \frac{Z_G + Z_1 + Z_2}{Z_4} + \frac{Z_G\, Z_1}{Z_5\, Z_6} + \frac{Z_2(Z_G + Z_1)}{Z_4\, Z_5} + \frac{Z_G(Z_1 + Z_2)}{Z_4\, Z_6}$$
$$+ \frac{Z_G(Z_1 + Z_2 + Z_3)}{Z_B\, Z_6} + \frac{(Z_G + Z_1)\,(Z_2 + Z_3)}{Z_B\, Z_5} + \frac{Z_3(Z_G + Z_1 + Z_2)}{Z_B\, Z_4} + \frac{Z_2 Z_3(Z_G + Z_1)}{Z_B\, Z_4\, Z_5} + \frac{Z_G Z_1(Z_2 + Z_3)}{Z_B\, Z_5\, Z_6}$$
$$+ \frac{Z_G Z_3(Z_1 + Z_2)}{Z_B\, Z_4\, Z_6} + \frac{Z_G\, Z_1\, Z_2}{Z_1\, Z_5\, Z_6} + \frac{Z_G\, Z_1\, Z_2\, Z_3}{Z_B\, Z_4\, Z_5\, Z_6}$$

For $\frac{E}{V}$ to be large, it is obvious that Z_B, Z_4, Z_5 and Z_6 should be small and Z_G, Z_1, Z_2 and Z_3 should be large.

APPENDIX V.

Solution of Low-Pass Filter networks having equal series chokes and equal shunt condensers.

In the following expressions given in Figs. **24/29**

G = inductance of generator
B = inductance of battery
L = inductance of each series choke
C = capacitance of each shunt condenser
E = alternating E.M.F. of generator
$\omega = 2\pi f$ where f is the frequency of the E.M.F.
V = P.D. across battery.

APPENDIX VI.

Design of a Low-Pass Filter type smoothing circuit for a typical generator.

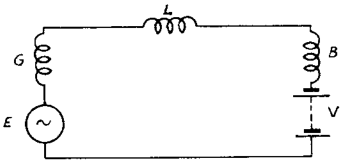
Details of generator:

Voltage, 25.
Full load current, 100 Amps.
Speed, 1500 R.P.M.
No. of armature slots, 37.

Details of battery:

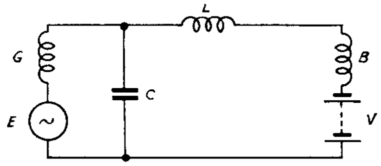
Inductance of circuit, 20 μH.
(calculated from proposed lay-out).

APPENDIX V (*continued*).



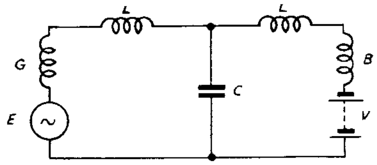
SINGLE CHOKE
FIG 24

$$\frac{E}{V} = 1 + \frac{L+G}{B}$$



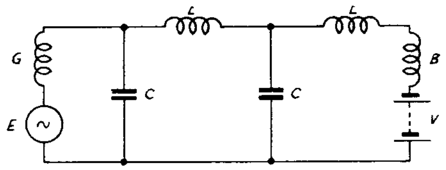
ONE SECTION
FIG 25

$$\frac{E}{V} = 1 + \frac{L+G}{A} - \frac{\omega^2 CG}{B}(L+B)$$



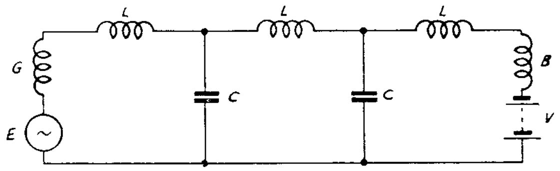
T SECTION
FIG 26

$$\frac{E}{V} = 1 + \frac{2L+G}{B} - \frac{\omega^2 C}{B}(L+G)(L+B)$$



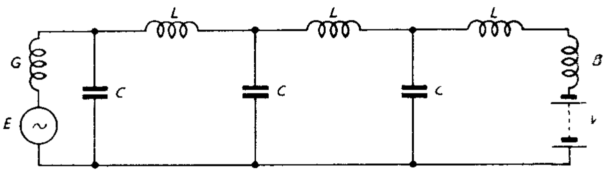
TWO SECTIONS
FIG 27

$$\frac{E}{V} = 1 + \frac{2L+G}{B} - \frac{\omega^2 C}{B}\left\{L(L+3G) + B(L+2G)\right\} + \frac{\omega^4 C^2 LG}{B}$$



TWO T SECTIONS
FIG 28

$$\frac{E}{V} = 1 + \frac{3L+G}{B} - \frac{\omega^2 C}{B}\left\{L(4L+3G) + B(3L+2G)\right\} + \frac{\omega^4 C^2 L}{B}(L+G)(L+B)$$



THREE SECTIONS
FIG 29

$$\frac{E}{V} = 1 + \frac{3L+G}{B} - \frac{\omega^2 C}{B}\left\{L(4L+6G) + B(3L+3G)\right\} + \frac{\omega^4 C^2 L}{B}\left\{L(L+5G) + B(L+4G)\right\} - \frac{\omega^6 C^3 L^2 G}{B}(L+B)$$

APPENDIX VI (*continued*).

Maximum P.D. across battery due to floating $= 0.5$ mV.

Noise E.M.F. of generator $= 0.01 \times 25$ V. $= 250$ mV.

∴ Required attenuation ratio of filter

$$= \frac{250}{0.5} = 500$$

For a generator of this rating a design factor of 4 should be used.

∴ Calculated attenuation ratio should be 2000.

Generator slot ripple frequency

$$= \frac{1500.\ 37}{60} = 925 \text{ c/s.}$$

$$\omega = 2\pi f = 5800$$

Generator self-inductance

$$= \frac{1{\cdot}4.\ 25}{100.\ 1500} \text{ Henries} = 233\ \mu\text{H.}$$

Using a single choke inductance L μH.

$$\frac{L + B + G}{B} = 2000$$

$$L = 2000.\ 20 - 233 - 20$$
$$= 40000 - 253 = 40\ m\text{H.}$$

This would be a very large and expensive choke, therefore try a single section filter (see Fig. **25**).

Try L $= 2\ m$H. and C $= 4000\ \mu$F.

$$\frac{E}{V} = \frac{2000 + 233 + 20}{20} - \frac{5800^2.\ 4000.\ 233\ (2000 + 233)}{20.\ 10^{12}}$$

$$= 113 - 3500$$
$$\doteqdot -3400$$

This is too large, therefore use C $= 3000\ \mu$F.

$$\frac{E}{V} = -2500 \text{ this is satisfactory.}$$

It may be more economical to use a T section filter, therefore try L $= 1\ m$H. and C $= 3000\ \mu$F.

$$\frac{E}{V} = 113 - \frac{5800^2.\ 3000.\ 1233.\ 1020}{20.\ 10^{12}}$$

$$= 113 - 6300 \doteqdot -6200 \text{ this is excessive.}$$

Try L $= 700\ \mu$H. and C $= 2000\ \mu$F.

$$\frac{E}{V} = \frac{1623}{20} - \frac{5800^2.\ 2000.\ 933.\ 720}{20.\ 10^{12}}$$

$$= 81 - 2260 = 2180 \text{ this would be satisfactory.}$$

It is probable that the total cost of the second filter (T section) would be less than that of the first. The total capacitance and also the total inductance is less. It must be remembered, however, that the voltage drop of each choke must be half the permissible total for the filter.

The resonance frequencies should be checked.

Single section L $= 2\ m$H. C $= 3000\ \mu$F.

$$f_r = \frac{1}{2\pi}\sqrt{\frac{1}{GC}} = \frac{1}{2\pi}\sqrt{\frac{10^{12}}{233.\ 3000}}$$

$$= \frac{1}{2\pi}\ \frac{10^3}{\sqrt{.7}} = 191 \text{ c/s}$$

this is satisfactory since $f_r < 200$ c/s.

T section. L $= 700\ \mu$H. C $= 2000\ \mu$F.

$$f_r = \frac{1}{2\pi}\sqrt{\frac{2}{LC}} = \frac{1}{2\pi}\sqrt{\frac{2.\ 10^{12}}{700.\ 2000}}$$
$$= 191 \text{ c/s.}$$

this is satisfactory.

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