No. 185

The Institution of Post Office Electrical Engineers.

The Operation of Valve Oscillators and their Synchronisation

A. C. LYNCH, M.A.
J. R. TILLMAN, Ph.D.
and
D. G. TUCKER, Ph.D., A.M.I.E.E.

A Paper read before the London Centre on 11th April, 1944.

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CONTENTS.

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PART 1.

THE DESIGN AND OPERATION OF OSCILLATORS.

This part of the paper considers the theory of oscillator driving circuits, mainly by interpreting them as circuits using positive feed-back; the alternative explanation, involving negative resistance, applicable to some circuits, is described briefly. Driving circuits are described whose stability is adequate for most present-day tuning circuits. The relative merits of "straight" and heterodyne oscillators are discussed. Some particular oscillators are described and discussed as examples of current practice.

1. Separation of Functions.

The use of thermionic valves as reliable circuit elements is now commonplace, and the complexity which can be tolerated in their circuits is increasing. This tendency to complexity has, however, led to the development of multi-valve oscillators in which the theory of operation is clearer than in the apparently simple types; for in the latter a single valve was, in fact, performing more than one function.

An oscillator circuit can be separated into two parts: (1) a maintaining circuit (more conveniently referred to as a "driving circuit"); and (2) a frequency-discriminating circuit (or "tuning circuit"). In a few special oscillators there are more than one of each of these circuits. The separation of these two functions is not complete, but is justified by its practical convenience. An example of an oscillator circuit showing this clear division is given in Fig. 1. The oscillator consists of a parallel resonant circuit coupled through resistors to an amplifier of flat frequency characteristic.

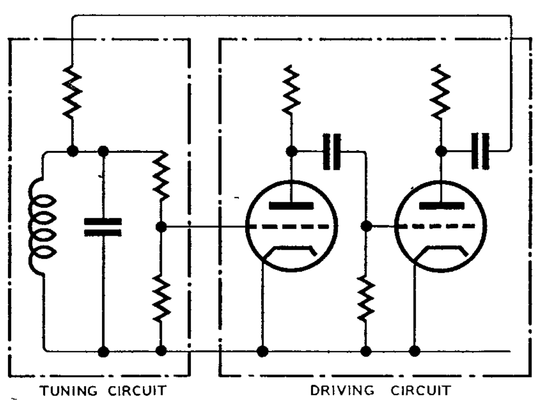

FIG. 1.—OSCILLATOR SHOWING SEPARATION OF FUNCTIONS.

It is convenient from a theoretical viewpoint to divide the driving circuit into a linear amplifier and a limiter. Up to the present time these two functions have not usually been physically separated, except in certain oscillators of high precision.

Modern oscillators, except those for very high frequencies, almost invariably use a separate amplifier to obtain the required power output, and this paper will not be concerned with questions of power output or efficiency of oscillator circuits.

The tuning circuit may be an oscillatory circuit, a mechanical resonator (such as a crystal, a tuning-fork, or a magnetostrictor), or a network of resistors and either capacitors or inductors. This paper is concerned mainly with showing how modern technique has developed driving circuits whose performance is better than that of most of the tuning circuits which are used with them. Its conclusions apply to oscillators using any type of tuning circuit.

2. Thermal Effects in Tuning Circuits.

All types of oscillator tuning circuit are liable to cause change of frequency through mechanical and thermal changes. The mechanical strength can usually be made suitably high by normal constructional methods. The thermal effects are of a fundamental nature, and can be overcome only by ingenuity of design.

Inductance and capacitance are both of the nature of lengths multiplied by a quantity which, for air-cored coils or air-dielectric condensers, may be taken as unity. They therefore tend to vary with temperature in the same way as the linear dimensions of the materials of which they are made; that is, by from 10 to 100 parts per million per degree Centigrade. Materials having various coefficients of expansion are available; and a suitable combination of different materials can result in the balancing of the effect of an expansion of one dimension by that of an expansion of another. For example, the increase in area of a condenser plate may be compensated by an increase in its separation from the opposite plate; the separation must increase proportionately twice as much as the linear dimensions of the plate.

Simple effects of this type, and methods of compensation based on them, rely on uniformity of temperature of all relevant parts of the apparatus. If the temperature is not uniform, compensation will fail, and mechanical distortion may occur, leading to very poor stability of the electrical properties. The conditions to be met in satisfactory design are that the time-constant of thermal equilibrium of each of the component parts of the apparatus should be similar and preferably short. As all convenient electrical insulating materials are also thermally insulating, this leads to the use of metal components insulated by small bushes—a method of construction which results in high electrical losses. In practice, construction mainly of metal is usual for condensers and single-turn coils, but is avoided for multi-turn coils because of the large eddy-current losses which would occur in their supports.

The use of materials whose permittivity or permeability varies with temperature gives a new degree of freedom in design. For precision work, however, such materials are best avoided: this temperature variation is often associated with high losses, and, especially in non-homogeneous materials such as iron-dust cores, the variation may not be cyclic. There are, however, some satisfactory ceramic materials whose permittivity increases by about 40 per million per °C. (There is, of course, no objection to the use of materials whose permittivity is constant with temperature—*e.g.*, polystyrene, transformer oil.)

The temperature coefficient of the frequency of a mechanical resonator (a crystal, a tuning-fork, or a magnetostrictor) is approximately that of its length, modified by that (usually much smaller) of its elasticity. A quartz crystal has a low thermal expansion, and the change in elasticity can be arranged (by choice of direction of cutting) to balance the expansion to within 1 per million per °C. A tuning-fork of Elinvar has a coefficient of about 10 per million per °C.

Thus no tuning circuit, except a quartz crystal, can be relied on permanently to vary less than, say, 5 per million per °C., except perhaps when the temperature variations are so slow that compensation can be fully effective. To obtain better results than this, some form of temperature control is required, a matter which is outside the scope of this paper. Driving circuits which enable an oscillator to attain a constancy within, say, 10 per million at constant temperature are therefore adequate for most present-day tuning circuits. In variable-frequency oscillators the resultant temperature coefficient of the tuning circuit is liable to be much greater in at least part of the range. The demand on the stability of the driving circuit is correspondingly less stringent.

3. Equivalence of Positive Feed-back and Negative Resistance.

Driving circuits may be classified as " positive feed-back " (*e.g.*, the Hartley, the Colpitts, or the Franklin circuits) and " negative resistance " (*e.g.*, the dynatron or the transitron) types. But suppose a resonant circuit to be connected to a network of unknown properties, and thereby to be maintained in oscillation: then the network may be interpreted as one of either type. The distinction is, at most, one of convenience. Both types of circuit supply power to the tuning circuit, and their instantaneous output is controlled by it.

The more general type is the positive feed-back circuit, whose property is that an applied potential E at a point P results in the development at another point Q of an amplified e.m.f. μE. If P and Q are connected through an impedance Z (whose value includes the output impedance of the amplifier), a current $\frac{(\mu - 1)\,E}{Z}$ flows to P, and the result at P is the same as if it had been connected to a resistance $-\frac{Z}{\mu - 1}$. A " negative resistance " possesses a similar property, but P and Q are already connected internally, and the impedance Z is not, in general, available as a separate entity to be selected at will.

General results can thus be obtained by consideration of the positive feed-back circuit, which is easily pictured, and it is in theory unnecessary to consider negative resistance oscillators separately. However, there are certain circuits, such as the dynatron, which can be more easily studied by using the conception of negative resistance.

4. Negative Resistance.

The negative resistance method of analysis is applicable to oscillators in which the tuning circuit is a two-terminal network: that is, it can be resolved into a simple parallel resonant circuit. The term " negative resistance " is applied to any device whose voltage-current characteristic, under certain conditions and over a limited range, has a negative slope (Fig. **2**).

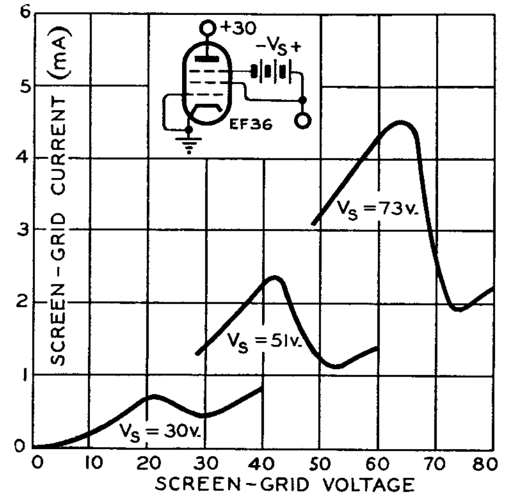

FIG. 2.—NEGATIVE RESISTANCE CHARACTERISTICS OF TRANSITRON MEASURED STATICALLY.

The term is not confined to those devices, such as dynatrons and transitrons, whose negative resistance characteristic can be measured by d.c. tests ; it can be applied to devices in which the characteristic is revealed only by a.c. tests. An example of the latter is a two-stage amplifier with output and input terminals connected together. If the relationship between current and voltage is known and can be accurately expressed algebraically both inside and outside the negative range, a full analysis of the oscillator is possible, although the mathematical processes involved (which include the solving of a differential equation with non-linear coefficients) are not simple.

Some interesting results have been obtained by this method.(1) The connection found between the frequency of oscillation and the harmonic content has been shown to agree with that derived by a more direct consideration of the influence of the harmonics. The method is important historically but the mathematical difficulties involved seem to have limited its use.

Oscillators normally use the negative resistance device connected in parallel with the resonant circuit. The amplitude of oscillation becomes such that at one or both peaks of the oscillation the resistance is large or even positive, and the average value of negative resistance during each cycle is equal to the positive resistance of the tuning circuit. The mathematical difficulty mentioned previously is that of determining what is meant by the " average " resistance over a cycle. The condition that oscillation shall occur is that the minimum value of the negative resistance is arithmetically less than the positive equivalent shunt resistance of the tuning circuit which it is to maintain.

The variation in negative resistance introduces harmonics into the waveform.

The shunt capacitance of several of these devices varies with the current flowing,[2] and is therefore changing during each cycle of oscillation. It is usually regarded as constant and as forming a part of the tuning circuit, but the variation should be allowed for in a complete investigation of these devices when used as oscillators; it will cause the generated wave to differ from a sine curve. The increase of input capacitance of a valve with increase of cathode current is well-known in practice, and is explained as a movement of the hypothetical " virtual cathode " formed under the influence of space-charge. In some circuits (*e.g.*, the dynatron) the valve capacitance is not that between grid and cathode, but is one of the other inter-electrode capacitances, which are, in general, more stable.

5. Positive Feed-back : Conditions for Steady Oscillation.

The condition for steady oscillation, expressed in terms of feed-back, is that if at some point the circuit is interrupted, then an oscillation originating at this point returns to it with the same amplitude and in the same phase after passing through the driving and tuning circuits.[3] For if not, the returning signal modifies the original one: a different amplitude causes the circulating signal to build up or to decay, while a phase shift tends to produce a wave of different frequency. The condition requires the gain in the driving circuit to be equal to the loss in the tuning circuit and the total of the phase shifts in the two circuits to be zero or a multiple of 2π.

The essential property of the tuning circuit is that it transmits different frequencies with different phase-shifts. The frequency of the oscillation, if any, may be imagined to adjust itself to satisfy the phase condition. Then if the gain of the driving circuit at that frequency is adequate, the circuit oscillates.

Since the driving circuit usually transmits a wide range of frequencies with little difference in phase-shift, the oscillation frequency is determined mainly by the tuning circuit. The foregoing argument shows, however, that it depends also on the driving circuit, and is not, in general, the same as the "natural frequency " of the tuning circuit.

The adjustment of driving circuit gain, as usually understood, to exact equality with tuning circuit loss would be impossibly difficult to maintain with normal circuits; yet without it a steady state cannot be reached. The adjustment is, of course, brought about by non-linearity of the amplifier; the ratio of output to input voltage must, at some value of output, fall. Overloading or non-linearity at some point in the circuit is therefore an essential feature of normal oscillator circuits.

6. Conditions of Good Stability of Frequency.

Assume a tuning circuit of very good stability, a condition not often realised except with a crystal. Then the stability of oscillation frequency depends on the constancy of the phase-shift in the driving circuit, and on the association of large phase-shift, ϕ, in the tuning circuit with small change of frequency. The latter property is characteristic of a circuit of high " Q " (Q being the ratio of the reactance of the inductor or capacitor to the total series resistance). In fact $d\phi/df$ is proportional to Q.*

This relation leads to several important conclusions. It shows the desirability of using high-Q circuits for stable oscillators of the inductance-capacitance type, and it is one of the two reasons for the excellent stability of a crystal oscillator. (The crystal has a very high equivalent Q, and also can be arranged to have a low temperature-coefficient.) For investigations into the relative merits of various driving circuits, tuning circuits of low Q should be used, so that frequency changes of the type now being discussed will outweigh thermal effects. In resistance-capacitance tuning circuits the value of $d\phi/df$ corresponds to a Q of the order of $\frac{1}{2}$, and frequency stability is therefore difficult to attain. There are oscillators in which the tuning circuit is a nearly-balanced Wien bridge[4] or a network of similar properties.[5] These circuits may be regarded as applying negative feed-back to the amplifier, and thereby reducing the driving-circuit phase-shift to a small fraction of what it would otherwise be. The magnitude of the feed-back is increased as the balanced condition is approached, the amplifier gain being simultaneously increased to maintain oscillation. Such oscillators can be made very stable. Their disadvantage is the need for a driving circuit of very high gain to overcome the large attenuation in the tuning circuit.

The phase-shift in the driving circuit should preferably be small, as $d\phi/df$ is at a maximum when ϕ is zero. If, for example, ϕ is $\pi/4$, then $d\phi/df$, and therefore the stability, fall to $\frac{1}{2}$ of their optimum value. This apparently trivial warning is ignored in one otherwise excellent circuit[6] in which ϕ tends towards $\pi/2$. When the condition $\phi = 0$ is satisfied, the oscillation frequency is the same as the natural frequency of an L-C tuning circuit. It is sometimes stated that the frequency is then independent of the properties of the driving circuit, and therefore stable. This statement is untrue.[7] It is true, however, that this condition gives the minimum frequency change for a given alteration of phase-shift in the driving circuit. It is also true that this condition makes the frequency nearly independent of the resistance of the tuning circuit, which changes considerably with temperature; this useful property appears to have been overlooked up to the present.

Alteration of the phase-shift in the driving circuit can arise in at least three ways: mechanical instability, causing changes in stray capacitance; change in valve capacitances; and changes in the magnitude of harmonics produced by overloading. The last is usually the most important, and is discussed in the following section of this paper.

The change of input capacitance of a valve not only produces harmonics as a result of change of capacitance within each cycle, as discussed above; it also causes a change of frequency if supply voltages vary, since the mean tuning capacitance then alters. As before, the grid-to-cathode capacitance is the least stable. Frequency changes arising directly from this

* See Appendix I.

cause can be prevented by stabilisation of power supplies.

Mechanical instability can cause frequency changes even when there is no apparent vibration. In an oscillator assembled on a steel panel with stiff wiring, changes of the order of 20 parts per million were stopped by tying certain of the wires to insulating supports.

7. Effect on Stability of Harmonic Production.

The overloading of a valve in the driving circuit produces harmonics which are fed into the tuning circuit. The harmonics are transmitted with a phase-shift different from that of the fundamental—usually $\pi/2$—and are also attenuated. Harmonic components, therefore, return to the overloaded valve. They then intermodulate with the fundamental, and one of the products is a wave of fundamental frequency whose phase differs by $\pi/2$ from that of the original wave and whose amplitude depends on the extent of the overloading. Combined with the original wave, it produces a wave whose phase-shift depends on the overloading.*[8] So also does the frequency of oscillation.

The presence of harmonics is thus not in itself a cause of frequency instability, which arises only when the magnitude of the out-of-phase intermodulation product changes; that is, only if the level or phase of the harmonics change, relative to the fundamental, and not always then. There are thus at least three directions in which increased stability can be sought: the reduction of a harmonic to a small value, its stabilisation at some definite value, and the shifting of its phase, relative to the fundamental, to a value of either zero or π. The two latter possibilities, which are both easy to realise in practice, have been ignored in previous accounts of oscillator theory; and published calculations[9] of the relation between stability and harmonic content, although generally accepted, do not apply to oscillators in which these latter devices are used. The false step in these calculations appears to be the assumption that the valve characteristic remains the same when, for some unspecified reason, the oscillation amplitude varies.

Four examples of stabilised systems follow.

(i) A group of tuned circuits may be used to reject harmonics.[10] This method, however, is cumbersome and scarcely applicable to variable-frequency oscillators.

(ii) The gain of the driving circuit may be so controlled that the amplitude of the oscillation remains small and constant, and no valve generates appreciable harmonics. There are two convenient methods available. In one,[11] a signal derived from the amplifier is rectified, smoothed, and used to control the gain of a valve in the amplifier circuit. (This is similar to the A.V.C. of a radio receiver.) The other method uses a bridge or potentiometer circuit[12] in which the resistance of one of the arms depends on the current flowing in it. The circuit can be so adjusted that the out-of-balance signal is of very constant amplitude.

Both circuits act as limiters without manufacturing appreciable harmonics, and the harmonic content will be fairly constant. Some care is necessary in oscillators of wide frequency-range to prevent "hunting" at a low frequency determined by the time-constant of the rectifier smoothing circuit or thermal element.

(iii) A driving circuit whose gain is stabilised by negative feed-back has the property that the relative intensities of the harmonics remain constant when the supply voltage changes. This is because, in any oscillator, the extent of limiting action required is determined solely by the amplifier gain and the tuning circuit loss. If the gain is stabilised, so also is the loss required in the limiter. The wave-form produced by the limiter under conditions of large feed-back is a sine-wave whose peaks, at one or both limits of the amplitude, are replaced by straight lines at certain limiting values. (Fig. 3). When the limiting occurs

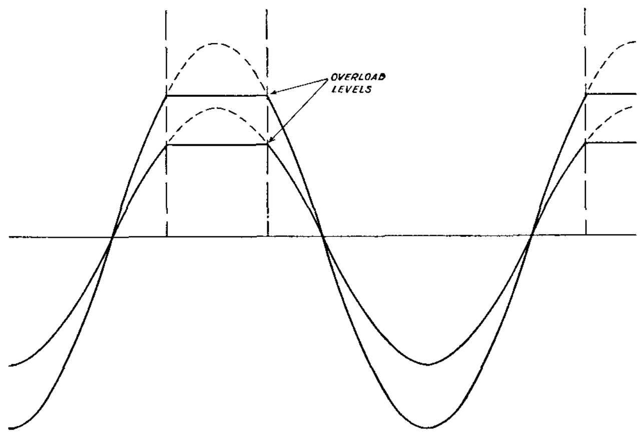

FIG. 3.—OUTPUT WAVES OF OVERLOADED FEED-BACK AMPLIFIER AT TWO OVERLOAD LEVELS.

at one peak only, there is a unique relation between the harmonic content and the ratio of fundamental output to fundamental input. When the supply voltage changes, the amplitude alters to that corresponding to the new cut-off point, but the shape remains the same. This is not true of amplifiers in which the gain depends on the supply voltage, since a change in gain requires a change in the output-to-input ratio of the limiter—that is, in the shape of the output wave.

(iv) The use of a second tuned circuit[13] (which need not be sharply tuned or accurately matched to the main one) subjects harmonics to additional attenuation and to an additional phase-shift of nearly $\pi/2$. They therefore return to the overloaded valve with a total phase-shift of nearly π, and any fundamental component generated by intermodulation is in opposite phase to the original wave and thus harmless.

Of these four systems, the third is much the simplest. Using this arrangement with reasonably steady power supplies (taken from a.c. mains) the

* See Appendix II.

frequency changes caused by the driving circuit are unimportant compared with those due to thermal and mechanical effects in inductance-capacitance tuned circuits. Frequency stability of 10 parts per million over several hours can be achieved in this manner, using a tuning circuit whose Q is 150. When advances in the design of tuning circuits justify it, it will be necessary to consider the relative practical merits of this system and of bridge-stabilization (the second system described). Until then, bridge-stabilization, although now coming into considerable use, is probably a needless complication except for oscillators of the highest precision—a class limited at present to crystal and tuning-fork oscillators.

8. Examples of "Straight" Oscillators.

The term " straight " is used, for lack of a better one, to mean oscillators of other than the heterodyne type discussed later. Straight oscillators find their greatest use at radio frequencies; in the audio range they are used for fixed frequencies, but rarely for variable frequencies on account of the awkward magnitudes of the tuning circuit components. Most of the radio-frequency oscillators are either crystal-controlled, when the interest attaches rather to the crystal than to its driving circuit, or are of a very crude type.

Two oscillators working in the carrier-frequency range will be described, and two working at lower frequencies.

A substandard wavemeter designed by a British firm[14] uses a dynatron and a parallel L-C network as the driving and tuning circuits respectively. (A dynatron is a tetrode valve in which the screen voltage exceeds the anode voltage by about 50. Since the secondary emission coefficient of the anode depends on the energy of the primary electrons, increase of anode voltages causes a decrease of anode current.) The range of frequency, covered by means of a set of interchangeable inductors, is, in one typical model, 30 to 10,000 kc/s. By using capacitors and inductors of special design and by specifying closely the working voltages of the dynatron valve, whose supplies are obtained from batteries, the makers have achieved good stability of frequency (10 parts per million over long periods under good laboratory conditions).

A carrier-frequency oscillator designed at the Post Office Research Station[15] is based on a two-stage pentode amplifier whose gain is about 52 db. without negative feed-back and 30 db. with it. The output is coupled through a suitable resistance to the tuning circuit, which consists of a dust-cored coil and a variable air-condenser with a small mica condenser to make the ratio of maximum to minimum frequency equal to about 2.6. Five combinations of tuning coil and coupling resistor are selected by a switch, and cover the range 10 kc/s to 1.2 Mc/s. The tuning circuit is connected to the amplifier input through a potentiometer consisting of condensers, including the input capacitance of the amplifier, giving very small phase-shift. The output is taken by another valve in parallel with the first of those of the oscillator proper, and the harmonic content at this point is of the order of $\frac{1}{4}$%. The stability of frequency varies over the frequency range: at frequencies up to 20 kc/s it is about 1 per million per 1% change of supply voltage, and at higher frequencies it is of the order of 5 per million per 1% change. Constancy within $\pm$ 1 db. of the output over the various ranges results from the feed-back method of limiting and from the use of tuning coils in which the effective shunt resistance remains fairly constant with frequency.

An audio-frequency oscillator, commercially available,[16] uses resistance-capacitance tuning circuits applying positive feed-back at the wanted frequency. The gain of the driving circuit is adjustable, and for the best results should be reset at each frequency. The tuning circuits contain resistances in parallel, calibrated in terms of the output frequency, which is read directly as the sum of those shown by four dials. The range is from 10 c/s to 100 kc/s, and the accuracy of frequency is of the order of 200 parts per million for all likely variations in temperature and in power supplies.

The mechanism of the Speaking Clock[17] is driven by a motor which is supplied with three-phase current at 4 c/s, generated by an oscillator and power amplifier. The oscillator consists of three similar amplifying valves coupled together cyclically. The frequency of oscillation is such that there is a phase-shift of $\pi/3$ in each coupling circuit, and is governed by the values of the coupling condenser and grid and anode resistors. The amplification of each stage is only slightly greater than unity, so that the waveform is good. The power supplies are stabilised. A small external synchronising signal is injected into one of the valves to control the frequency very accurately.

9. Principles of the Heterodyne Oscillator.

The heterodyne oscillator, which is of considerable importance practically, consists essentially of two similar oscillators and a means of obtaining from them a frequency equal to the difference of their frequencies. It involves no additional problems of a fundamental nature, but there are many minor points worthy of attention in the design.

The main advantages of the heterodyne arrangement for variable-frequency oscillators include the wide range of frequency obtainable with a single control, and the comparative constancy of output as the frequency is changed. It can more easily be made to give low harmonic content; and in an audio-frequency oscillator the inductors and capacitors are of more convenient sizes and can often be made to have higher " Q " and hence give better frequency stability. The only inherent disadvantage is the greater complication, although a badly-designed or badly-maintained oscillator may produce signals of several spurious frequencies mixed with the wanted one. The frequency stability and permanence of calibration, compared with those of a " straight " oscillator using components of similar quality, can be of the same order; but they are in general inferior,* except for the effect of the higher Q.

The high-frequency or "carrier" oscillators may be of any type giving adequate stability. They should

* See Appendix III.

be of similar design, so that changes of power supplies and of ambient temperature will produce approximately proportional changes of frequency in each. They should be sufficiently well screened from each other to prevent any tendency to locking when their frequencies are nearly the same, as otherwise the wave-form at low frequencies will be poor. The effect on the frequency calibration of small permanent changes in the " constants " of either tuning circuit is much reduced if, as is usual, a zero beat control is incorporated. This control enables the beat frequency at zero scale reading to be adjusted to zero; it usually takes the form of a small variable condenser placed in parallel with the main tuning condenser. The magnitude of the errors remaining after use of this control is discussed in Appendix III.

The variable-frequency oscillator usually has its maximum frequency, rather than its minimum, equal to the fixed frequency, as the components used in the two tuning circuits are then more nearly alike. The fixed frequency may then be of any value greater than twice the highest wanted frequency. If it is too near this limit, the performance necessary in the low-pass filter is difficult to obtain. If it is too high, the beat frequency, which is the small difference of two large quantities, becomes unduly sensitive to small differences between the properties of the two oscillators. A fixed frequency of from five to ten times the highest wanted frequency is usually a reasonable choice.

The mixing device[18] will in general produce from the two oscillator frequencies, f_0 and $f_0 - f$, a series of frequencies given by the general formula $pf_0 \pm q\,(f_0 - f)$, where p and q are integers or zero. This series includes some high frequencies which are easily removed by a low-pass filter; the wanted frequency f; also harmonics of f, which cannot be filtered out. Some mixers, however, would in theory not give rise to any terms for which the sum of p and q exceeds 2. In practice, such mixers give these terms at low levels, and there are then no unwanted frequencies at appreciable levels within the range of frequencies required from the oscillator. A valve working on its anode-bend characteristic, which usually approximates to a parabola, approaches this condition. So can a multi-electrode valve in which the signals are applied to two grids.

The presence of harmonics in the high-frequency supplies will result in the generation of harmonics of the wanted frequency. If one of the high frequencies is free from harmonic, however, this cannot occur. It is easy to reduce the harmonics of the fixed high frequency to very low levels, and this precaution, coupled with the use of one of the above mixers, is sufficient to produce an output nearly free from harmonic over the whole range of wanted frequencies.

The level of the output signal depends on the levels of the two signals reaching the mixer. The range of the variable-frequency oscillator is so small a proportion of the frequency that this oscillator can easily be set up to give not only good wave-form, but also substantially constant output, over this range. The beat frequency level can therefore be made substantially constant with frequency.

10. Examples of Heterodyne Oscillators.

Current practice will be illustrated by descriptions of four oscillators designed wholly or partly by Post Office engineers.

The first of these, known as the Oscillator No. 12,[19] was designed fundamentally about ten years ago and has had extensive use. Fig. 4 (*a*) shows the design

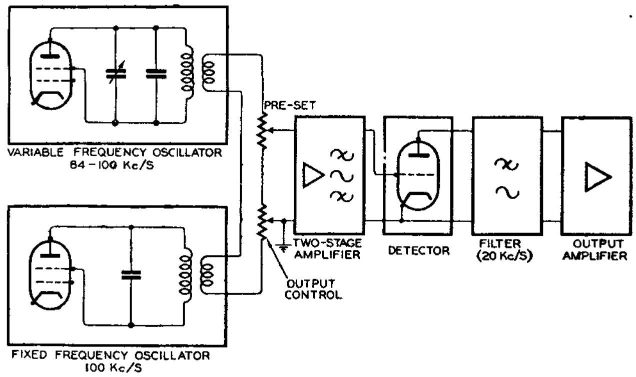

FIG. 4 (*a*).—OSCILLATOR NO. 12 (RYALL-SULLIVAN).

schematically. Dynatrons are used in the two high-frequency oscillators. The signals obtained from two pick-up coils, coupled respectively to the tuning coils of the oscillators, are added, amplified, and filtered to remove harmonics of the high frequencies. The resulting signal is rectified in an anode bend detector, and the beat frequency signal is subsequently amplified without distortion. A harmonic compensator is used to reduce the already small level of the signal of frequency $2f$, which, originally produced by the detector, appears in the output. Great care has been taken in the design and construction of the components of the tuned circuits in order to maintain stability with time and with temperature changes.

The second oscillator was originally a modification, in respect of frequency range, of the No. 12, but today its design[20] (Fig. 4 (*b*)) differs at several points

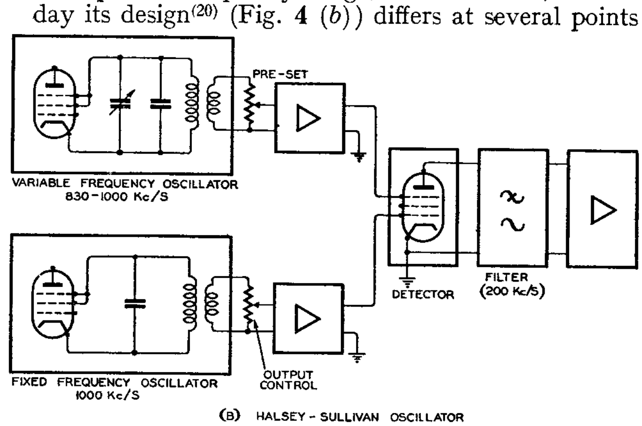

FIG. 4 (*b*).—HALSEY-SULLIVAN OSCILLATOR.

from the earlier type. Transitron oscillators generate the two high-frequency signals. Pick-up coils are used as before to feed the two amplitude controls (one of which is normally pre-set), but the signals are separately amplified. The amplifier used for the fixed frequency is sharply tuned to 1 Mc/s, the other much more broadly to 800—1000 kc/s. The two amplified sinusoidal signals are injected separately to the control grid and suppressor grid of a pentode, and a low-pass filter follows this detector.

In the third oscillator (Fig. **4** (*c*))[21] each high-

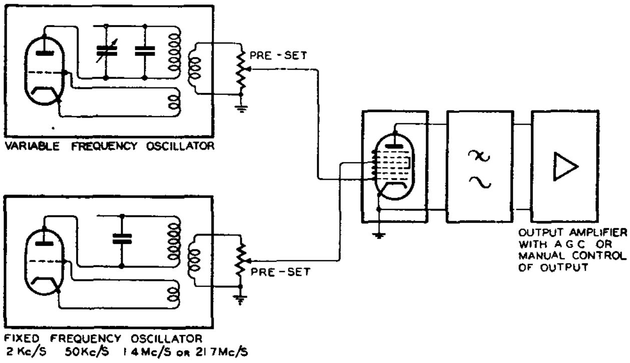

FIG. 4 (*c*).

frequency oscillator consists of a triode having regenerative inductive coupling between its anode and grid circuits. Fractions of the signals appearing in the two anode circuits are applied to an octode frequency changer, one to each control grid. By means of switches, beat frequency ranges of 10—100 c/s, 100—3000 c/s, 3—150 kc/s, and 150—3000 kc/s can be selected. Unlike the first two oscillators described, this was not intended to be in the precision class.

The fourth oscillator[22] has been designed to meet a special requirement and has a beat frequency range 0.1 (or less) to 65 c/s. The design (Fig. **4** (*d*)) could

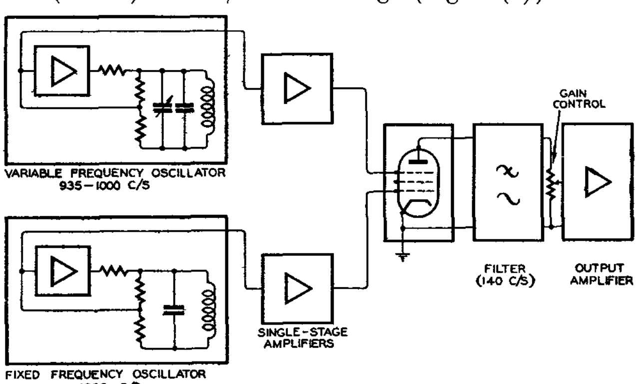

FIG. 4 (*d*).

be applied to other ranges, however. The two high-frequency signals, produced by oscillators of the two-stage amplifier type and having low harmonic content, are amplified separately. The detector resembles that used in the second example. Despite the low value of the " high " frequency, the beat frequency range is still covered by the variation of an air condenser forming part of the tuning circuit of the variable frequency oscillator.

The differences in design of these four units are considerable. Some of the merits and weaknesses of the driving circuits are discussed elsewhere in this paper. The dynatron, depending as it does for its negative resistance on the rather unstable value of the coefficient of secondary emission of the inner wall of the anode, is a less reliable circuit element from this point of view than the transitron; but its stability has proved adequate in practice. It has the advantage that its contribution to the capacitance of the tuning circuit is relatively stable with time and with change of power supplies. That of the transitron is subject to considerably larger changes. The triode oscillator of the third example is probably unsuitable for use in precision oscillators unless precautions are taken to stabilise its power supplies. The negative resistance of the driving circuit of the last example depends for its stability largely on that of several resistors and has an effective capacitance which, since its main component is only a fraction of the control-grid-to-cathode capacitance of a pentode, is small and fairly constant.

Pick-up coils, if rigidly mounted, provide a satisfactory method of extracting signals from the tuning circuits. Each coil should, however, be terminated by an impedance of very high or constant value, a condition not satisfied by the first two examples, in which the output control shunts the pick-up coils of the fixed frequency oscillator. The effect of the input admittance (largely capacitive) of the amplifier shunting the control on the " constants " of the tuning circuit of the oscillator varies with the position of the control. The reduction of the resulting variation of beat frequency with setting of the output control to a negligible amount has been achieved only by severely limiting the number of turns on the pick-up coils. This step has necessitated an additional stage of amplification at the carrier frequencies. In the circuits of both the third and fourth examples the output control follows so much later that this weakness of design is avoided. The addition of the two high-frequency signals as carried out in the first example is not to be recommended if good wave-form of the beat frequency signal is desired at very low frequencies, because it increases the likelihood of pull-in and near pull-in. On the other hand, if a detector which does not require separate injection is to be used, this procedure requires the use of only one high-frequency amplifier and filter.

The use of multi-electrode valves as frequency changers was unknown when the first oscillator was designed. As used in the last three examples, this detector avoids any large coupling between the two high-frequency oscillators. If both the signal to the first control grid and that to the second control grid are small enough to maintain a linear relationship between the anode current and the levels of these two signals, the output of harmonics is very small, and the level of beat frequency obtained is proportional to the product of the two levels. Small changes which occur in the level of the variable frequency signal as its frequency is varied will therefore appear in the level of the beat frequency ; suitable compensation is, fortunately, easily obtained. If the variable-frequency signal is made large, causing a switching action in the valve, the output signal is independent of the level of this large signal; but the harmonic production is higher. Similar effects occur in the anode bend detector, according as the signals do or do not remain within the parabolic part of its characteristic. In the No. 12 oscillator a high level of variable-frequency signal is used, and the output is constant with frequency.

The use of a low-pass filter and output amplifier is common to all the examples.

PART II.

THE SYNCHRONISATION OF OSCILLATORS, WITH PARTICULAR REFERENCE TO CARRIER TELEPHONE SYSTEMS.

1. Definitions : Synchronism, Isochronism, and Frequency Regulation.

At the outset it is as well to define our use of the term "synchronisation." Strictly, perhaps, the term should only be applied to a system in which one signal is maintained identical in frequency and phase to another, controlling, signal. But in reference to oscillators it is more convenient to apply the term to the case where the frequencies are maintained identical although the relative phase angle may vary over a certain range. "Isochronism" is a better term for this condition, but "synchronism" has been in use so long that it is best to retain it. Another term with the same meaning is "locking." A synchronised oscillator is often referred to as a "locked oscillator."

It is important to distinguish between synchronisation and frequency regulation. The latter does not involve identity of controlled and controlling frequencies, but merely an approximate equality.

2. Application to Carrier Telephone Systems.

Post Office engineers will be interested in synchronisation and frequency regulation chiefly from the point of view of their application to carrier telephone systems, in which it is now considered desirable to maintain a close degree of equality between input and output signal frequencies. If the frequency received from a carrier channel is not the same as that applied, then difficulties are experienced with telegraph and dialling systems, and distortion is caused to speech. It is thus necessary to control the demodulating carrier frequency to be approximately equal to the modulating frequency.

Two methods of control have been applied; these are, as defined above, synchronisation and frequency regulation.

3. The Synchronisation of a Simple Oscillator by Injection.

In all carrier systems used by the Post Office for telephony, the carrier frequencies are obtained basically from a valve oscillator. If, then, the oscillator at the receiving end of a system is controlled by that at the sending end, the necessary synchronisation or frequency regulation is obtained. An oscillator may be most conveniently synchronised by the direct injection of the control frequency into its circuit. The control frequency will normally be transmitted from the sending (or controlling) end to the receiving (or controlled) end over a working telephone line.

3.1 *Simple Theory of Synchronisation.**

The mechanism by which an injected signal is able to synchronise an oscillator is entirely a function of the non-linear properties of the oscillator circuit. Part 1 of this paper has made clear that all oscillators contain at least one element having a characteristic which varies with amplitude, without which their operation would be impracticable. Consider the simple feed-back oscillator of Fig. 5, in which the

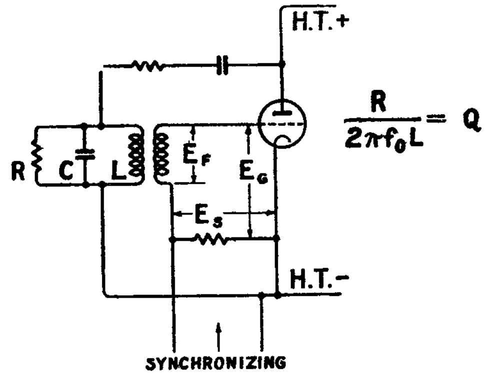

FIG. 5.—FEED-BACK OSCILLATOR.

valve serves as both maintaining and non-linear elements. If a tone of frequency f_s is injected into the grid circuit, where f_s is very near the natural oscillation frequency f_0, then there is a large amount of positive feed-back to this frequency, and the amplification to the frequency f_s is in consequence very high. Thus a very small input voltage may result in a large voltage on the grid. The presence of this voltage may so reduce the amplification of the valve (owing to its non-linearity) that the natural oscillation can no longer take place, and only the "forced" oscillation at frequency f_s remains. Provided this is stable, the oscillator is now said to be synchronised, since its output frequency is identical with the control frequency.

Fig. 6 shows a vector diagram of the voltages now concerned in the grid circuit. E_F is the voltage fed

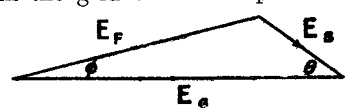

FIG. 6.—VECTOR DIAGRAM FOR SYNCHRONISED OSCILLATOR.

back via the tuned circuit, E_S is the injected synchronising voltage, and E_G is the resultant grid voltage. For equilibrium, the condition is evidently that the vector sum of E_F and E_S must equal E_G. Let the angle between E_S and E_G be θ. Let also the grid amplitude of the natural oscillation (in the absence of the injected signal) be E_{G0}. Then it can be shown[29] that there are two conditions determining whether or not the free oscillation will be suppressed and the forced oscillation be stable, *i.e.*, whether or not the oscillator will remain synchronised. These conditions are that, for synchronisation,

$$E_G > \frac{1}{\sqrt{2}} E_{G0}^* \qquad \text{.................................(1)}$$

and

$$-\frac{\pi}{2} < \theta < \frac{\pi}{2} \qquad \text{.................................(2)}$$

* See Bibliography, references 23-29, for a list of original literature on this subject.

* Strictly speaking, this condition applies exactly only if the nonlinear law is a cubic.

Synchronisation will fail when either of these conditions becomes an equality. It can be shown that the first then applies for large injected voltages, the second for small. In practice, only the second is likely to be encountered.

Thus synchronisation may be said to fail when the phase angle between the injected signal and the grid voltage becomes 90°. This condition is known as pull-out, and will be dealt with again later. Pull-in is, of course, the converse phenomenon.

3.2 *The Chief Properties of a Synchronised Oscillator.*

It is thus seen that the phase angle θ between controlled and controlling signals may vary over a range of $\pm$ 90° without pull-out. The manner in which the angle varies between one limit and the other is given by the relation

$$\sin \theta = \frac{E_G}{E_{G1}} \cdot \frac{f_0 - f_s}{f_0 - f_{s1}} \qquad (3)$$

where the suffix "1" indicates the value at pull-out. If the oscillator is working very non-linearly, or if the injected voltage is very small, E_G and E_{G1} are very nearly equal, and the graph of θ against frequency difference $(f_0 - f_s)$ is a sine curve, as shown in Fig. **7**.

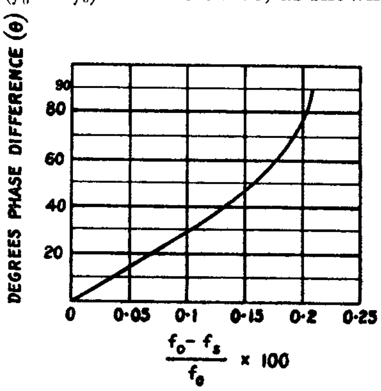

Fig. 7.—Variation of Phase Angle.

In general, the output voltage of the oscillator is not constant over the synchronising frequency range. It is a maximum approximately when the applied and natural frequencies are identical, and a minimum at pull-out. It is difficult to define this variation by a simple mathematical expression, but Fig. **8** illustrates

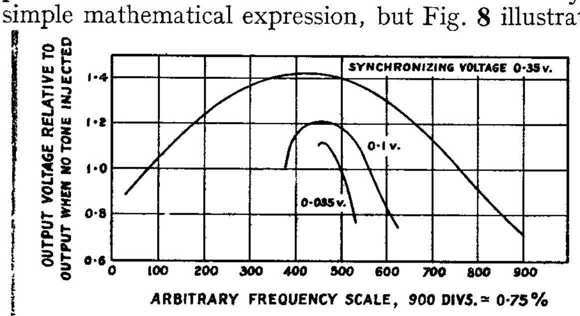

Fig. 8.—Variation of Output Voltage.

it with some actual observed values. The variation is smaller with small values of injected voltage, or with large natural oscillation amplitudes (*i.e.*, with a large degree of non-linearity). The lack of symmetry which is quite noticeable is due to the shift of natural frequency caused by harmonics, which was described in Part 1 of this paper.

A most important relationship is readily deduced from the vector diagram of Fig. **6**. At pull-out, E_{S1} is perpendicular to E_G. If ϕ is the angle between E_F and E_G, then at pull-out

$$\frac{E_{S1}}{E_{G1}} = \tan \phi \qquad (4)$$

where the suffixes "1" are again used to indicate pull-out values. Now, if the tuned circuit of the oscillator has a value of shunt resistance-to-reactance ratio equal to Q, then it is readily shown that near resonance,

$$\tan \phi = 2\,Q\,(f_0 - f_{s1})/f_0 \qquad (5)$$

assuming f_0 is the resonant frequency of the tuned circuit as well as the natural frequency of the oscillator.

Therefore

$$E_{s1} = 2\,Q \cdot E_{G1}\,(f_0 - f_{s1})/f_0 \qquad (6)$$

This equation enables the required injected voltage to be calculated for any given oscillator and for any required range of synchronised frequencies. In practice, the natural frequency will drift, and enough controlling signal must be injected to ensure that synchronism is maintained over the whole range of drift. The synchronised frequency range is seen to be proportional to the injected voltage.

When pull-out does occur, the natural oscillation is not, in general, recommenced, but beats set in at a frequency lower than the difference between natural frequency and injected frequency. Typical beats are shown in Fig. **9**; when the oscillator is only just pulling out, they are very slow, and appear as a sudden slip of one cycle in between two periods of synchronism. This condition is known as "near pull-in" and is often evident in heterodyne oscillators when the difference frequency is small and the two main oscillators tend to pull-in to one another. As the oscillator is detuned more and more, the beats become more rapid, and eventually the beat frequency is equal to the difference between natural and injected frequencies.

4. Indirect Method using Injection and Frequency Division.[30]

The simple system of direct synchronisation described above has been applied to Post Office Carrier Systems Nos. 2 and 4. The actual carrier frequencies have been transmitted on the line, and used to synchronise the carrier oscillators at the distant station. But with Carrier System No. 5 (the original 12-circuit system) such a procedure is not possible. Here the channel carriers (12 in number) are generated from a 4 kc/s master oscillator, so that it is this oscillator which needs to be synchronised. One way of doing this is to transmit a 4 kc/s tone to line from the oscillator at one station, and to use this to synchronise the oscillator at the distant station. But this is not generally feasible, as 12-circuit routes are not designed to transmit 4 kc/s. A much better system, which has been adopted for the temporary synchronisation of No. 5 routes, is to transmit to line the 15th harmonic (60 kc/s) of the control oscillator. At the controlled station this is modulated with the

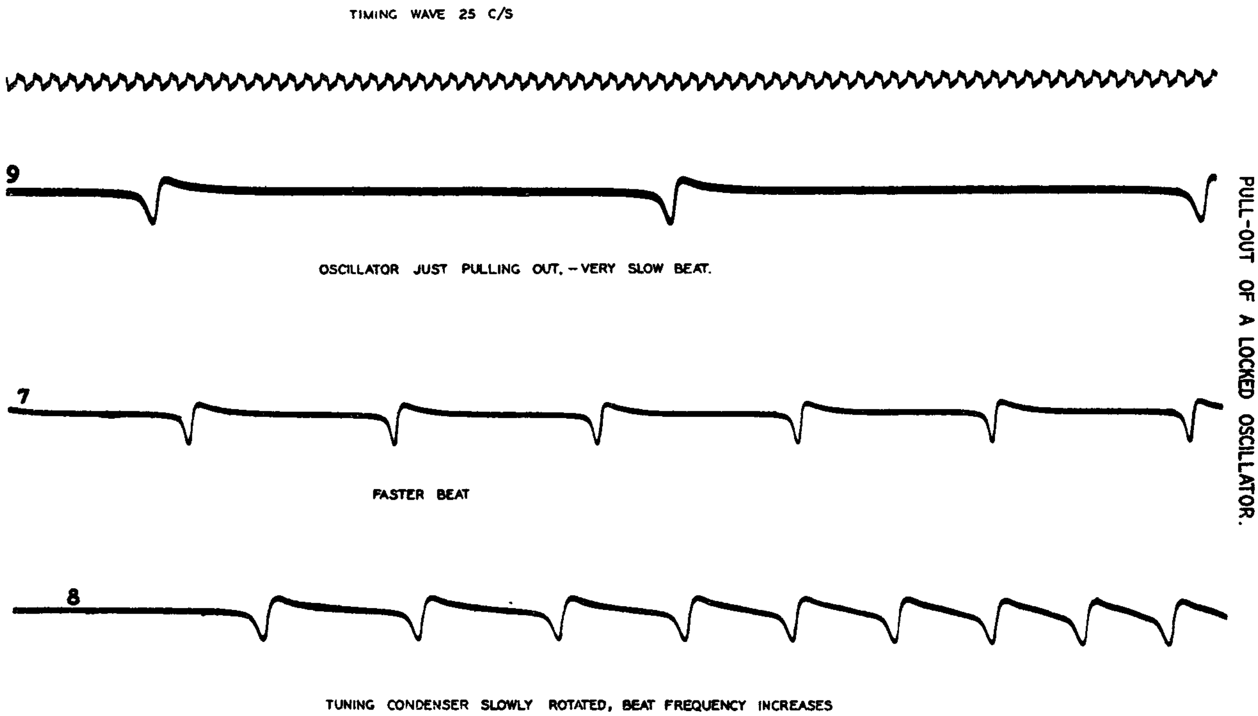

FIG. 9.—PULL-OUT OF A LOCKED OSCILLATOR.

14th harmonic (nominally 56 kc/s) of the local oscillator. The resultant frequency of nominally 4 kc/s is injected into the oscillator, and it is readily seen that if the oscillator has an initial error of δ c/s, then the injected frequency has an error of $-14\ \delta$ c/s. The result is that as the injected frequency tends to synchronise the oscillator, the error δ becomes zero, and exact synchronisation is automatically achieved. The general schematic of this system is shown in Fig. **10**. One advantage it has is that phase variations

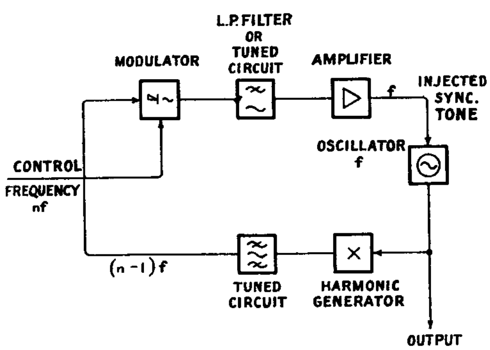

FIG. 10.—SYNCHRONISING SCHEME USING HIGH CONTROL FREQUENCY.

are only $\pm 90°$ between pull-out limits on the top (60 kc/s) channels and only $\pm 6°$ on the oscillator itself. In the direct system using a 4 kc/s control tone, the variations would be $\pm 90°$ on the oscillator and $\pm 1350°$ on the top channel.

A natural development of this system is to omit the oscillator, and to obtain a frequency divider,[30, 31] as shown in Fig. **11**, which is incapable of free oscillation

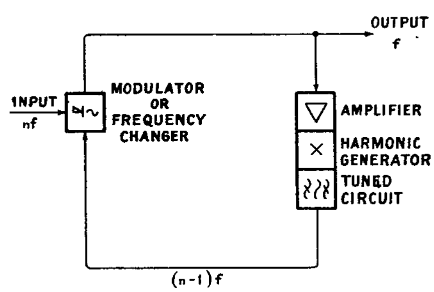

FIG. 11.—FREQUENCY DIVIDER.

or pull-out, and which has very small phase-variations. This circuit is now quite well-known as the "regenerative modulator," and need not be described further here.

5. The Objects of retaining the Oscillator in the System.

It should be pointed out that, of course, it is not really necessary to use a synchronised oscillator in a carrier system, since the transmitted control tone could be filtered out and used direct. This would have two disadvantages, however;

(*a*) the filtration might be very difficult, as the tone used at the controlled station must be very nearly pure. A synchronised oscillator has a very considerable discriminating effect against unwanted frequencies, even when these are extremely close to the wanted frequency,[32] and

(*b*) if the control tone should fail for any reason, there would be no longer any carrier supply at the controlled station. With the use of oscillators, it would still be possible to work the carrier system, the oscillator frequency being adjusted manually from time to time.

Thus, in carrier systems, oscillators are used practically without exception.

6. Frequency Regulation.

A system of frequency regulation has been incorporated in Carrier System No. 7. This method involves the use of a small induction motor to drive the fine tuning condenser of the 4 kc/s oscillator. The motor has applied to it a rotating field produced from a phase-splitting of the beat frequency between the control and oscillator frequencies. Its rotor will thus rotate when this beat frequency is not zero, and it is arranged to move in such a direction that its effect on the tuning of the oscillator causes a decrease in the beat frequency. Eventually the frequency error becomes zero, and the rotor becomes stationary. The system has been more fully described in an article in the P.O.E.E.J.[33]

The chief advantages of this system, as compared with the injection systems described above, are:—

(*a*) If the control tone fails, the oscillator remains at the frequency at which it was last set, and does not immediately jump to a different frequency.

(*b*) The inertia of the system is high, and interference which may be present with the control tone is practically completely eliminated. It should be noted, though, that the discrimination of the injection system is readily made high enough for all existing carrier systems.

The chief disadvantages are:—

(*a*) A considerable time is required to effect frequency adjustment, and if the frequency is continually needing adjustment, noticeable frequency errors may arise.

(*b*) There is no control of phase, so that there is no limit to the phase drifts which may occur.

(*c*) The system is very much more complicated, with consequent difficulties of maintenance.

In conclusion, it must be emphasized that this account of synchronisation of oscillators is only a summary of a very large subject. Fuller details can be obtained from the literature listed in the Bibliography, particularly reference No. 29.

APPENDIX I.

RELATION BETWEEN PHASE SHIFT AND CHANGE OF FREQUENCY.

Consider a parallel resonant circuit composed of a capacitance C and an inductance L, the resistance R of the inductor being such that $Q = \frac{\omega L}{R}$ is constant.

Then the impedance of the circuit

$$= \frac{R + j\omega L}{1 - \omega^2 LC + j\omega CR}$$

and therefore the phase angle ϕ is given by

$$\tan\phi = \frac{\omega L(1 - \omega^2 LC) - \omega CR^2}{R(1 - \omega^2 LC) + \omega^2 LCR}$$

$$= Q(1 - \omega^2 LC) - \frac{\omega^2 LC}{Q}$$

$$\frac{d\phi}{d\omega} = \frac{\dfrac{d\tan\phi}{d\omega}}{\dfrac{d\tan\phi}{d\phi}} = \frac{-2\omega LC\left(Q + \dfrac{1}{Q}\right)}{1 + \tan^2\phi}$$

If $\phi = 0$ and Q is large, this becomes

$$\frac{d\phi}{d\omega} = -2\omega LCQ = -\frac{2Q}{\omega}.$$

or

$$\frac{d\phi}{df} = -\frac{2Q}{f}$$

With the above assumption that Q is large, $\phi = 0$ corresponds to $\omega^2 LC = 1$. But if ϕ has some value other than zero or a multiple of 2π, it can be shown that $\frac{d\phi}{df}$ has a smaller value; for example, if $\phi = \frac{\pi}{4}$, *i.e.*, $\tan\phi = 1$, and Q is large

then $\frac{d\phi}{d\omega} = -\omega LCQ \risingdotseq -\frac{Q}{\omega}$

or $\frac{d\phi}{df} = -\frac{Q}{f}$

APPENDIX II.

PHASE SHIFT OF FUNDAMENTAL PRODUCED BY INTERMODULATION OF HARMONICS.

As a simple case, consider a non-linear element whose characteristic is given by $V = aV_0 + bV_0^2$ where V_0 and V are the input and output voltages respectively.

Let a voltage $E \sin \omega t$ be supplied to this element.

Then the output

$$= aE\sin\omega t + bE^2\sin^2\omega t$$

$$= E\left\{a\sin\omega t + \frac{bE}{2}(1 - \cos 2\omega t)\right\} \qquad \text{..........(1)}$$

Since harmonics are subjected to a phase shift of nearly $\frac{\pi}{2}$ and the direct-current component is blocked at some stage, the wave returning to the modulator is approximately of the form

$$E \left\{ c \sin \omega t - d \cos \left(2\omega t + \frac{\pi}{2} \right) \right\},$$

$$\textit{i.e.,}\ E \{ c \sin \omega t + d \sin 2\omega t \}$$

where $\frac{d}{c}$ depends on the relative attenuations of the frequencies in the tuning circuit. This gives rise to an output $aE\,(c \sin \omega t + d \sin 2\omega t) + bE^2\,(c^2 \sin^2 \omega t + 2cd \sin \omega t \sin 2\omega t + d^2 \sin^2 2\omega t)$....................(2) of which one term can be reduced to

$$bE^2 \cdot cd\,(\cos \omega t - \cos 3\omega t)$$

That is, it contains a term of fundamental frequency but of phase $\frac{\pi}{2}$ different from that of the original input wave.

The complete effect of the repeated circulation of the signal round the network is the sum of a rapidly converging series, and the equations (1) and (2) are the sums to 1 and 2 terms respectively. Further terms are of no interest—they do not affect the fact that an out-of-phase signal of fundamental frequency is present.

APPENDIX III.

Stability of Frequency and of Frequency Calibration of a Heterodyne Oscillator.

For the sake of simplicity, the resonant circuit of each high-frequency oscillator will be assumed to consist of one inductor only, shunted by one or more capacitors, but similar considerations will be found to apply to more complex tuning circuits.

Then the maintenance of the accuracy of the frequency calibration depends primarily on (1) the stability of the frequency f_0, of the fixed frequency oscillator; (2) the stability of the inductance in the variable frequency oscillator; and (3) the stability of the capacitance of the variable condenser, which is directly calibrated in terms of the beat frequency. The zero beat control can be used to compensate fully for changes occurring in the fixed tuning condenser of the variable frequency oscillator.

If f_0 becomes $f_0\,(1 + a)$ and the zero beat control is correctly adjusted, the beat frequency corresponding to a setting f of the beat frequency control will be in error by

$$af \left\{ \frac{f_0^2 + f_0 f_1 + f_1^2}{f_0^2} \right\}$$

(writing f_1 for the variable high frequency).

If $f_0 \gg f$ this becomes $3\,af$, *i.e.*, the error in the beat frequency is, proportionally, 3 times that of the fixed frequency oscillator.

If the fixed frequency oscillator remains extremely stable or its frequency can be readjusted to its original value, but the inductor of the variable frequency oscillator changes in value by a factor b, and the zero beat control is correctly readjusted, the beat frequency will be in error at any setting f by

$$bf \left\{ \frac{f_0 f_1 + f_1^2}{2 f_0^2} \right\}$$

For values of f small compared with f_0, this represents twice the proportional change of frequency of a straight oscillator employing extremely stable capacitors but the same inductor as here.

If the main frequency control changes by a factor c, the beat frequency will be in error by an amount

$$cf \left(\frac{f_0 f_1 + f_1^2}{2 f_0^2} \right).$$

When $f_0 \gg f$ this is twice the proportional error in the variable frequency, but the comparison is not very useful because a straight oscillator would not employ fixed and variable condensers in the same proportions.

In practice, over long periods, a combination of the above three changes will take place.

The provision of a second frequency check,(34) at a frequency near the highest required, is claimed to increase the calibration accuracy by a factor of 10. The control used must change the frequencies of both oscillators simultaneously so that the zero beat setting is unaffected.

The stability of a heterodyne oscillator over a period of a few hours is governed by the temperature coefficients of the two tuning circuits of the high-frequency oscillators, provided that the two oscillators are themselves well designed. Thus if the fixed frequency oscillator has a temperature coefficient of frequency of a, the variable one of b, the beat frequency, for a setting of f, will change by

$$t\,\{bf + (a - b)\,f_0\}$$

when the temperature of each tuning circuit rises by t. The term $t(a - b)f_0$ manifests itself as an error even at zero frequency* and can be eliminated by readjustment of the zero beat control. If the rise in temperature differs for the two tuning circuits, the change of beat frequency will, in general, be much greater. It will be seen that when the temperature rises are equal, and $a = b$, the temperature coefficient of the beat frequency equals that of either high-frequency oscillator.

The utmost precautions must be taken therefore when designing and manufacturing the components of the tuning circuits of a heterodyne oscillator intended to be in the precision class to ensure that they have, separately, the highest possible stability.

The short period stability is largely a function of the changes of frequency of each high-frequency oscillator with changes of supply voltages. In general the beat frequency will change at the same rate as, or at a greater than, the high-frequency oscillators. Once again it is not sufficient that the two high-frequency oscillators should show equal proportional changes; each should have the highest stability possible if the beat frequency is to be stable.

* This assumes that the temperature coefficient b applies equally to the inductor and the fixed condenser of the variable frequency oscillator and to the variable condenser. If the last component has a different coefficient, as well it may, a more complicated expression is obtained for the change of beat frequency ; some of this change may still be eliminated if resetting of the zero beat control can be made.

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